

Algebra 1 - Exponential Functions and Growth - Answer Key

Common Core F-LE.A.1, F-LE.A.2, F-LE.A.3, F-IF.C.7e, F-IF.C.8b · maths-pdf.fr/en/

Use this key to check your work on the exponential functions worksheet. Every solution shows the steps, and the final answer appears in bold. If your answer differs, find the first line where your work and the key disagree, then redo the problem from there without looking.

Algebra 1 · Common Core State Standards: **F-LE.A.1, F-LE.A.2, F-LE.A.3, F-IF.C.7e, F-IF.C.8b**

Exercise 1 - Identify a and b

In $y = a \cdot b^x$, $a = 3$ and $b = 2$. Since $b > 1$, it is growth.

Starting value 3; factor 2; growth

Exercise 2 - Decay factor

$b = 0.8$, and $1 - 0.8 = 0.2$.

Factor 0.8; 20% decrease

Exercise 3 - Complete a table

x	0	1	2	3
y	4	12	36	108

Each value is 3 times the one before.

$4 \cdot 3^0 = 4$, $4 \cdot 3^1 = 12$, $4 \cdot 3^2 = 36$, $4 \cdot 3^3 = 108$.

4, 12, 36, 108

Exercise 4 - Exponential table

Ratios: $\frac{6}{2} = 3$, $\frac{18}{6} = 3$, $\frac{54}{18} = 3$.

Exponential with ratio 3

Exercise 5 - Linear table

Differences: $9 - 5 = 4$, $13 - 9 = 4$, $17 - 13 = 4$. The ratios are not constant ($\frac{9}{5}$, $\frac{13}{9}$).

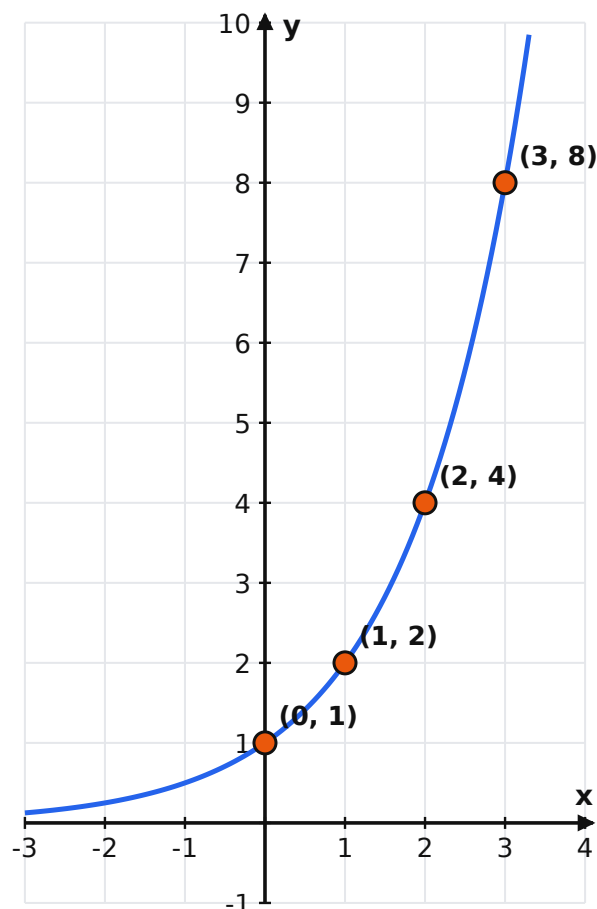
Linear, adding 4 each step

Exercise 6 - Write from a table

The starting value is $a = 5$ and each step doubles: $b = 2$.

$$y = 5 \cdot 2^x$$

Exercise 7 - Graph a growth function



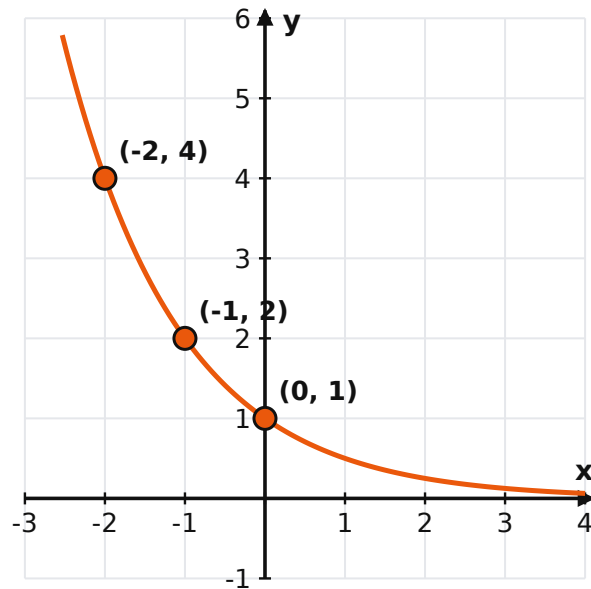
Points and curve.

Compute $2^{-2} = \frac{1}{4}$, $2^{-1} = \frac{1}{2}$, $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$.

Plot the points and draw a smooth curve that approaches the x-axis on the left.

Curve through (0, 1), (1, 2), (2, 4), (3, 8)

Exercise 8 - Graph a decay function



Decay curve.

Values: $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$.

The curve falls from left to right and gets close to the x-axis.

Decreasing curve through (0, 1)

Exercise 9 - Rate to factor

(a) $1 + 0.12 = 1.12$. (b) $1 - 0.08 = 0.92$.

1.12 and 0.92

Exercise 10 - Evaluate with a calculator

$$1.05^{10} \approx 1.62889.$$

$$100(1.62889) \approx 162.89.$$

162.89

Exercise 11 - Intercept and asymptote

At $x=0$, $y=1$. Any real number can be an exponent, and the output is always positive. As x decreases, 2^x gets closer to 0.

(0, 1); all real numbers; $y>0$; approaches the x-axis

Exercise 12 - Solve an exponential equation

$$32 = 2^5, \text{ so } x = 5.$$

$$81 = 3^4, \text{ so } x = 4.$$

$$x = 5 \text{ and } x = 4$$

Exercise 13 - Linear versus exponential

x	2	6	10
$f(x) = 10x + 5$	25	65	105
$g(x) = 2^x$	4	64	1024

Values of both functions.

$$f(10) = 10(10) + 5 = 105 \text{ and } g(10) = 2^{10} = 1024.$$

$f(10) = 105$; $g(10) = 1024$; g is larger

Exercise 14 - Domain and range

Any real number can be an exponent, so the domain is all real numbers.

The output is always positive, so the range is $y > 0$.

Domain: all real numbers; range: $y > 0$

Exercise 15 - Rewrite the base

$$2^{3t} = (2^3)^t = 8^t.$$

$$y = 8^t; \text{ factor } 8$$

Exercise 16 - Town population

$$\text{Growth factor } 1 + 0.04 = 1.04: P = 20000(1.04)^t.$$

$$P = 20000(1.04)^5 \approx 20000(1.21665) \approx 24333.$$

$$P = 20000(1.04)^t; \text{ about } 24,333 \text{ people}$$

Exercise 17 - Car value

$$\text{Decay factor } 1 - 0.15 = 0.85: V = 24000(0.85)^t.$$

$$V = 24000(0.85)^3 = 24000(0.614125) = 14739.$$

$$V = 24000(0.85)^t; \text{ \$14,739}$$

Exercise 18 - Compound interest

$$A = 1500(1.06)^8.$$

$$1.06^8 \approx 1.593848, \text{ so } A \approx 1500(1.593848) = 2390.77.$$

$$\text{\$2,390.77}$$

Exercise 19 - Medicine half-life

$$\text{Substitute } t = 18: A = 80\left(\frac{1}{2}\right)^3.$$

$$= 80 \cdot \frac{1}{8} = 10.$$

10 mg

Exercise 20 - Viral video

Day 2: $50 \cdot 3^2 = 450$ and $50 + 200(2) = 450$ (equal).

Day 3: $50 \cdot 3^3 = 1350$ and $50 + 200(3) = 650$.

From day 3 on, Video A is ahead and the gap grows.

Equal on day 2; A is larger from day 3

Exercise 21 - Doubling time

Solve $2000 \cdot 2^{t/12} = 8000$: $2^{t/12} = 4 = 2^2$.

$\frac{t}{12} = 2$, so $t = 24$.

24 years