

Algebra 1 - Solving Quadratic Equations - Lesson

Common Core A-REI.B.4a, A-REI.B.4b, A-SSE.B.3a, A-CED.A.1 · maths-pdf.fr/en/

A quadratic equation has the form $ax^2 + bx + c = 0$. Solve it by taking square roots, by factoring with the zero product property, by completing the square, or with the quadratic formula. The discriminant $b^2 - 4ac$ tells whether there are two, one, or no real solutions.

Algebra 1 · Common Core State Standards: **A-REI.B.4a, A-REI.B.4b, A-SSE.B.3a, A-CED.A.1**

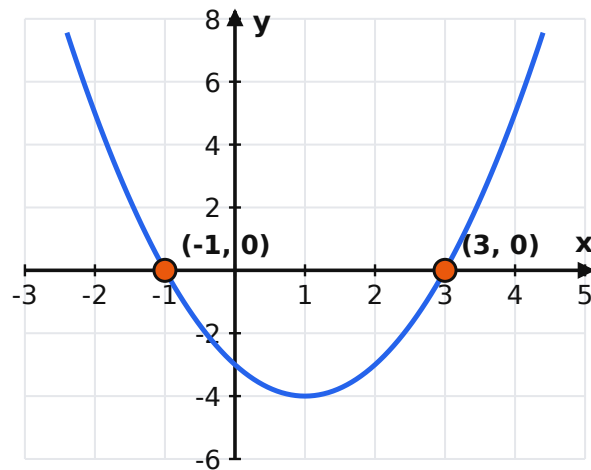
Key facts

- Standard form: $ax^2 + bx + c = 0$ with $a \neq 0$.
- Zero product property: if $A \cdot B = 0$, then $A = 0$ or $B = 0$.
- Square roots: $x^2 = k$ gives $x = \pm\sqrt{k}$.
- Quadratic formula: $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$.
- The discriminant $b^2 - 4ac$: positive for two real solutions, zero for one, negative for none.

What makes an equation quadratic?

Quadratic equation

An equation that can be arranged as $ax^2 + bx + c = 0$ with a not zero is **quadratic**. Its solutions, called **roots** or zeros, are the places where the parabola $y = ax^2 + bx + c$ meets the x-axis.



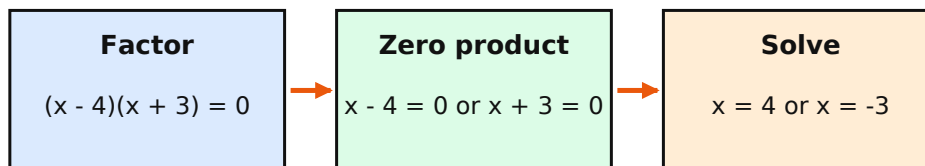
The solutions of $x^2 - 2x - 3 = 0$ are the x-intercepts -1 and 3 .

When can you use square roots?

If the squared term stands alone, as in $x^2 = k$, take the square root of both sides and keep both the positive and the negative result. For $x^2 = 20$ this gives $x = \pm\sqrt{20} = \pm 2\sqrt{5}$.

How does the zero product property help?

Move every term to one side, factor, and use this fact: when $A \cdot B = 0$, at least one of the factors must be zero, that is $A = 0$ or $B = 0$.



Factoring and the zero product property.

1. Get zero on one side.
2. Factor the other side.
3. Set each factor equal to zero and solve each small equation.
4. Substitute your answers into the original equation to verify them.

What does completing the square do?

It rewrites the equation so that one side is a perfect square. Starting from $x^2 + 6x = 7$, add 9, because half of the x-coefficient, squared, is $\left(\frac{6}{2}\right)^2 = 9$. The left side becomes $(x+3)^2 = 16$, so $x+3 = \pm 4$, and finally $x = 1$ or $x = -7$.

	x	3
x	x^2	$3x$
3	$3x$	9

Completing the square: x^2+6x plus the corner 9 makes the square $(x+3)^2$.

When is the quadratic formula the best choice?

It works for every quadratic. For $ax^2+bx+c=0$, the solutions are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, where the radicand b^2-4ac is called the **discriminant**.

Discriminant $b^2 - 4ac$	Number of real solutions
positive	two
zero	one
negative	none

The discriminant tells how many real solutions a quadratic has.

Example: To solve $x^2+4x+1=0$, read off $a=1, b=4, c=1$. The discriminant is $16-4=12$, which gives $x = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$.

In applications such as areas and falling objects, test each answer against the story. A negative length or time is not acceptable.

Questions about solving quadratic equations

What are the solutions of a quadratic equation?

They are the values of x that make the equation true. They are also the x -intercepts of the graph, and they are called roots or zeros.

When should you use the quadratic formula?

Use it when the quadratic does not factor easily, or when you want a method that always works. Write the equation in standard form to identify a , b , and c first.

What is the discriminant?

It is $b^2 - 4ac$, the expression under the square root in the quadratic formula. Positive means two real solutions, zero means one, and negative means no real solutions.

Why do you get two solutions when you take a square root?

Both a positive and a negative number square to the same value. For $x^2 = 9$, both 3 and -3 work.

How do you complete the square?

Add the square of half the coefficient of x to both sides. For $x^2 + 6x$, add 9 to get $(x + 3)^2$.