

Algebra 1 - Graphing Quadratic Functions and Transformations - Lesson

Common Core F-IF.C.7a, F-IF.C.8a, F-BF.B.3, A-SSE.B.3b · maths-pdf.fr/en/

A quadratic function $f(x) = ax^2 + bx + c$ has a U-shaped graph called a parabola. The vertex lies on the axis of symmetry $x = -b/2a$. In vertex form $f(x) = a(x - h)^2 + k$, the vertex is (h, k) . Changing k , h , and a shifts, stretches, or reflects the graph.

Algebra 1 · Common Core State Standards: **F-IF.C.7a, F-IF.C.8a, F-BF.B.3, A-SSE.B.3b**

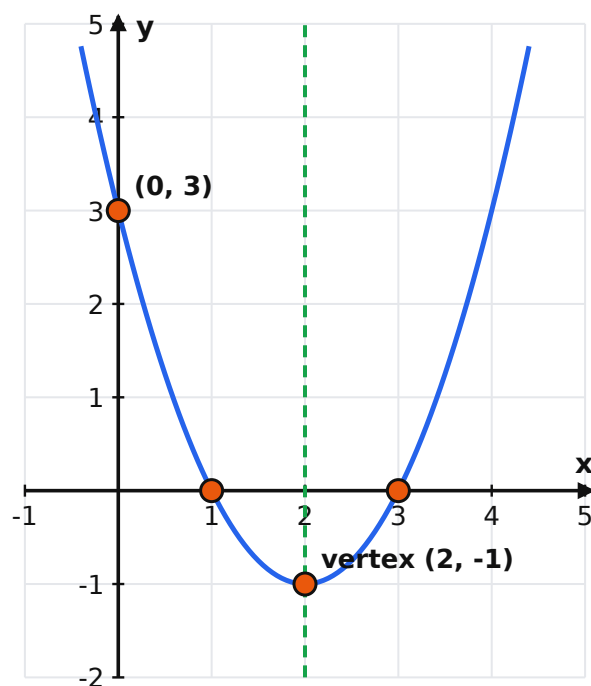
Key facts

- Standard form $f(x) = ax^2 + bx + c$; vertex form $f(x) = a(x - h)^2 + k$.
- The axis of symmetry is $x = -b/(2a)$, and the vertex lies on it.
- If $a > 0$ the parabola opens up (minimum); if $a < 0$ it opens down (maximum).
- $f(x) + k$ shifts up/down; $f(x - h)$ shifts right/left; $a \cdot f(x)$ stretches or reflects.
- The vertex gives the maximum or minimum value in a model.

What does the graph of a quadratic function look like?

Quadratic function and parabola

A **quadratic function** has the form $f(x) = ax^2 + bx + c$ with a not 0. Its graph is a **parabola**. The **vertex** is the highest point (maximum) if a is negative, or the lowest point (minimum) if a is positive. The **axis of symmetry** is the vertical line through the vertex.



A parabola has a vertex, an axis of symmetry (dashed), x-intercepts, and a y-intercept.

How do you find the vertex and intercepts?

1. The axis of symmetry is $x = -\frac{b}{2a}$. The vertex has this x-coordinate.
2. Substitute that x-value to find the y-coordinate of the vertex.
3. The y-intercept is $(0, c)$. Find x-intercepts by factoring or the quadratic formula.
4. Plot the vertex, the intercepts, and symmetric points, then draw a smooth curve.

Example: For $y = x^2 - 4x + 3$, the axis is $x = 2$, the vertex is $(2, -1)$, the y-intercept is 3, and the x-intercepts are 1 and 3.

Form	Equation	What you see
Standard	$y = ax^2 + bx + c$	y-intercept c
Vertex	$y = a(x - h)^2 + k$	vertex (h, k)
Factored	$y = a(x - p)(x - q)$	x-intercepts p, q

Each form of a quadratic shows a different feature.

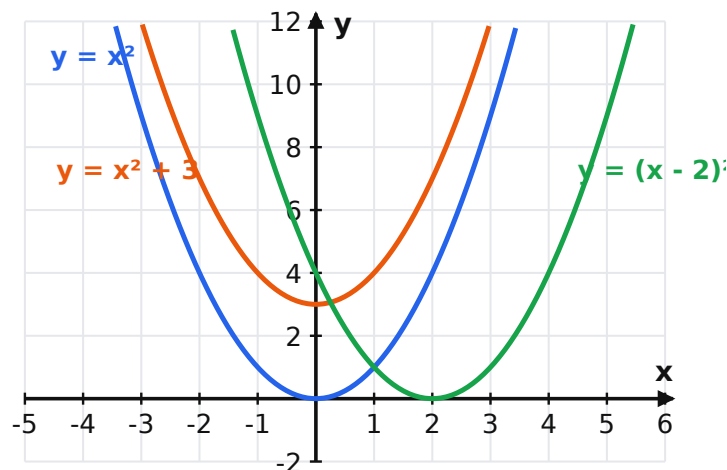
What does vertex form tell you?

In $f(x) = a(x - h)^2 + k$, the vertex is (h, k) , the axis is $x = h$, and a controls the width and direction. Completing the square turns standard form into vertex form: $x^2 + 6x + 5$ becomes

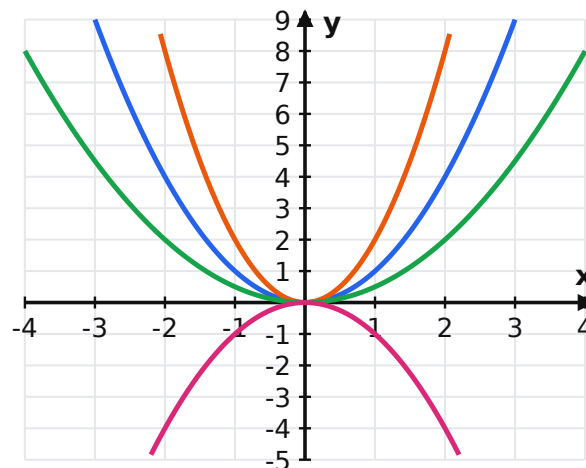
$$(x+3)^2-4.$$

How do transformations change the graph?

Starting from $f(x) = x^2$: adding k shifts the graph up (positive) or down (negative); replacing x by $x-h$ shifts it right when h is positive; multiplying by a stretches it, and a negative a reflects it over the x-axis.



Adding 3 shifts the parabola up. Replacing x with $x-2$ shifts it right 2.



A factor a greater than 1 narrows the parabola, a factor between 0 and 1 widens it, and a negative a flips it.

For a quadratic model, the vertex gives the best value. If $R = -5p^2 + 100p$ is revenue, the vertex (10,500) gives the price that earns the most.

Tip: To sketch a parabola quickly, plot the vertex first, then two points on each side at equal distances from the axis of symmetry.

Questions about graphing quadratic functions and transformations

How do you find the vertex of a parabola?

Use $x = -b/(2a)$ for the x-coordinate, then substitute into the function for the y-coordinate. For $y = x^2 - 4x + 3$, $x = 2$ and $y = -1$.

What is the axis of symmetry?

It is the vertical line through the vertex, $x = -b/(2a)$. The two halves of the parabola are mirror images across it.

How do you know if a parabola opens up or down?

Look at the sign of a . If a is positive it opens up; if a is negative it opens down.

What does $(x - 2)^2$ do to the graph of $y = x^2$?

It shifts the graph 2 units to the right. A common mistake is to shift left because of the minus sign.

How can you find the maximum of a quadratic model?

Find the vertex. If the parabola opens down, the y-coordinate of the vertex is the maximum value.