

Algebra 1 - Arithmetic and Geometric Sequences

- Lesson

Common Core F-IF.A.3, F-BF.A.1a, F-BF.A.2, F-LE.A.2 · maths-pdf.fr/en/

A sequence is an ordered list of numbers that can be written as a function of the position n . In an arithmetic sequence you add a common difference d each time. In a geometric sequence you multiply by a common ratio r each time. Recursive rules build each term from the previous one.

Algebra 1 · Common Core State Standards: **F-IF.A.3, F-BF.A.1a, F-BF.A.2, F-LE.A.2**

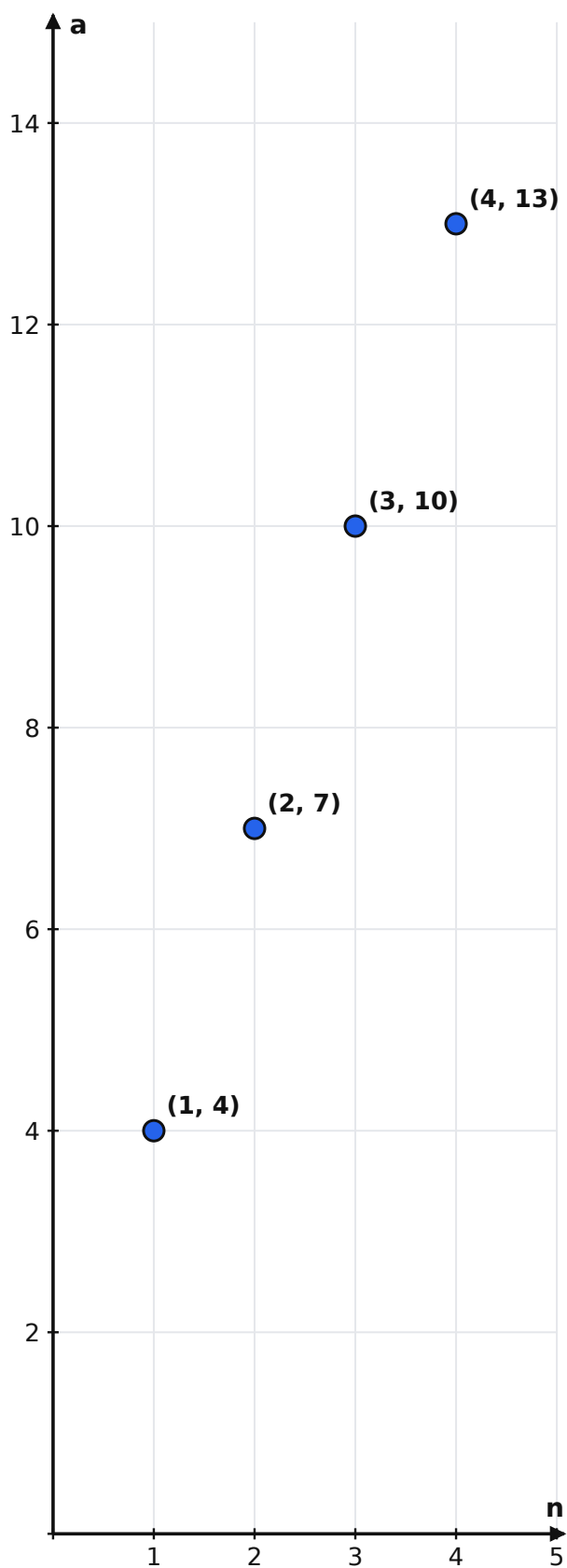
Key facts

- A sequence is a function whose inputs are the positions 1, 2, 3, and so on.
- Arithmetic: add d each time; $a_n = a_1 + (n - 1)d$.
- Geometric: multiply by r each time; $a_n = a_1 \cdot r^{(n-1)}$.
- A recursive rule gives each term from the previous term; an explicit rule gives any term directly.
- Arithmetic sequences are linear; geometric sequences are exponential.

What is a sequence?

Sequence

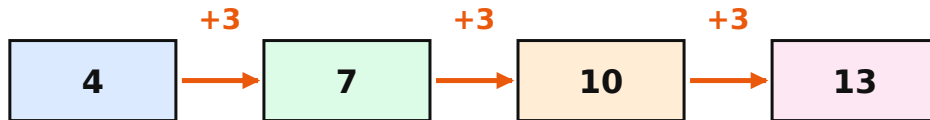
A **sequence** is an ordered list of numbers called **terms**. It is a function whose inputs are the position numbers $n = 1, 2, 3$, and so on, and whose outputs are the terms a_n .



The sequence $a_n = 3n + 1$ is a function whose graph is a set of separate points on a line.

What is an arithmetic sequence?

In an **arithmetic sequence**, you add the same number, the **common difference** d , to get the next term.

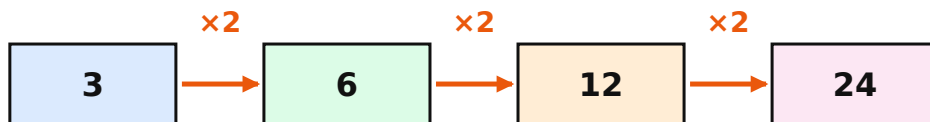


Arithmetic sequence: add the same number each time (common difference 3).

The **recursive rule** says how to get a term from the previous one: $a_n = a_{n-1} + d$. The **explicit rule** gives any term directly: $a_n = a_1 + (n-1)d$. For 4, 7, 10, 13 this is $a_n = 4 + (n-1)(3)$, which simplifies to $a_n = 3n + 1$.

What is a geometric sequence?

In a **geometric sequence**, you multiply by the same number, the **common ratio** r , each time.



Geometric sequence: multiply by the same number each time (common ratio 2).

The recursive rule is $a_n = r \cdot a_{n-1}$ and the explicit rule is $a_n = a_1 \cdot r^{n-1}$. For 3, 6, 12, 24 the explicit rule is $a_n = 3 \cdot 2^{n-1}$, so the 8th term is $3 \cdot 2^7 = 384$.

How do you write a rule for a sequence?

1. Subtract consecutive terms. If the differences match, it is arithmetic.
2. Divide consecutive terms. If the ratios match, it is geometric.
3. Substitute the first term and the common difference or ratio into the explicit rule.
4. Check the rule with the first two terms.



Each figure adds 2 squares: $a_n = 2n + 3$.

Example: The figures use 5, 7, and 9 squares. The differences are 2, so the rule is $a_n = 5 + (n-1)(2)$, which simplifies to $a_n = 2n + 3$. Figure 10 uses $2(10) + 3 = 23$ squares.

Arithmetic sequences grow by equal steps, so they are linear. Geometric sequences grow by equal factors, so they are exponential.

Tip: Check a rule with the first two terms before you use it. For $a_n = 3n + 1$, the first term is $3(1) + 1 = 4$ and the second is $3(2) + 1 = 7$, which match the list.

Questions about arithmetic and geometric sequences

How do you find the common difference?

Subtract any term from the next term. In 4, 7, 10, 13 the common difference is $7 - 4 = 3$.

How do you find the common ratio?

Divide any term by the term before it. In 3, 6, 12, 24 the common ratio is $6 \div 3 = 2$.

What is the difference between recursive and explicit rules?

A recursive rule needs the previous term, such as $a_n = a_{n-1} + 3$. An explicit rule, such as $a_n = 3n + 1$, gives the n th term directly.

Why is a sequence a function?

Each position number n has exactly one term a_n , so the sequence assigns one output to each input from 1, 2, 3, and so on.

How can you tell arithmetic from geometric?

If consecutive terms have a constant difference, it is arithmetic. If they have a constant ratio, it is geometric.