

Algebra 1 - Slope and Rate of Change - Lesson

Common Core F-IF.B.4, F-IF.B.6, F-LE.A.1b · maths-pdf.fr/en/

Slope measures how steep a line is: it is the rise divided by the run between any two points. In a real situation, slope is a rate of change with units, such as miles per hour. For a function that is not linear, the average rate of change over an interval is the slope between its endpoints.

Algebra 1 · Common Core State Standards: **F-IF.B.4, F-IF.B.6, F-LE.A.1b**

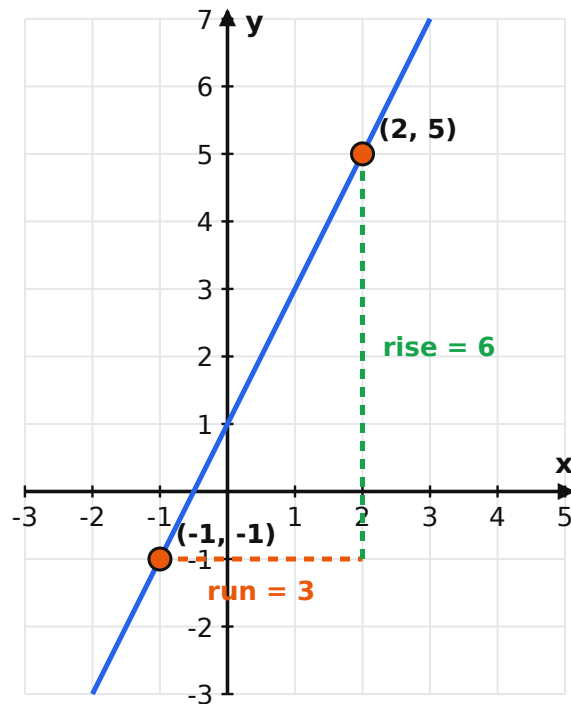
Key facts

- Slope = rise $\div$ run = $(y_2 - y_1)/(x_2 - x_1)$.
- Positive slope rises, negative slope falls, horizontal lines have slope 0, vertical lines are undefined.
- In a context, slope is a rate of change with units like dollars per hour.
- A linear function has the same rate of change between any two points.
- Average rate of change over $[a, b]$ is $(f(b) - f(a))/(b - a)$.

What is slope?

Slope

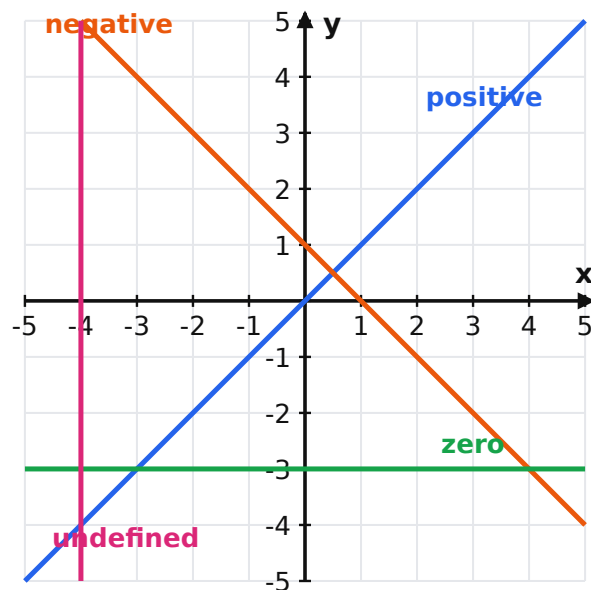
The **slope** of a line is the ratio of the vertical change to the horizontal change between two points: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$. It tells how fast y changes for each 1 unit of x .



From $(-1, -1)$ to $(2, 5)$: rise 6, run 3, so the slope is $\frac{6}{3} = 2$.

Pick two points on the line and count the **rise** (up or down) and the **run** (right). Here the rise is 6 and the run is 3, so the slope is 2.

What do positive, negative, zero, and undefined slopes mean?



Positive slope rises, negative slope falls, a horizontal line has slope 0, a vertical line has undefined slope.

A line that goes up from left to right has positive slope. A line that goes down has negative slope. A horizontal line has slope 0 because the rise is 0. A vertical line has an undefined slope because the run is 0 and you cannot divide by 0.

How is slope a rate of change?

In a real situation, slope is a rate of change with units. In the table, a car travels 110 miles every 2 hours, so the rate is $\frac{110 \text{ miles}}{2 \text{ hours}}$, which equals 55 miles per hour. A linear function has the same rate of change between any two points.

Hours x	Miles y
0	0
2	110
4	220
6	330

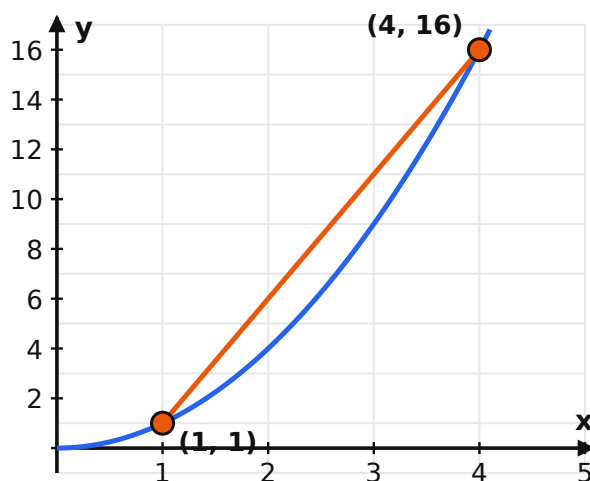
Every 2 hours the distance grows by 110 miles, so the rate is 55 miles per hour.

1. Choose two points (x_1, y_1) and (x_2, y_2) .
2. Subtract the y-values for the rise and the x-values for the run in the same order.
3. Divide rise by run and reduce.
4. Attach units if the situation has them.

Example: Find the slope through $(2, 3)$ and $(6, 11)$. The rise is $11 - 3 = 8$ and the run is $6 - 2 = 4$. The slope is $\frac{8}{4} = 2$.

What is average rate of change?

For a function that is not linear, the rate changes. The **average rate of change** from a to b is $\frac{f(b) - f(a)}{b - a}$, the slope of the segment joining the two points on the graph.



The average rate of change of $f(x) = x^2$ from 1 to 4 is the slope of the segment: $\frac{16-1}{4-1} = 5$.

In context, the intercept 15 in $y = 15 + 8x$ is the starting value when x is 0, and the slope 8 is the rate of change.

Questions about slope and rate of change

How do you find the slope between two points?

Subtract the y-values to get the rise and subtract the x-values in the same order to get the run, then divide. The points (2, 3) and (6, 11) give $8/4 = 2$.

Why is the slope of a vertical line undefined?

Every point has the same x-value, so the run is 0. Division by 0 is undefined.

What does slope mean in a word problem?

It is the rate of change: how much the output changes for each 1-unit increase in the input, with units such as dollars per hour.

How can you tell from a table that a function is linear?

If the x-values increase by equal steps and the y-values change by the same amount each step, the rate of change is constant and the function is linear.

What is average rate of change?

It is the change in the output divided by the change in the input over an interval. It equals the slope of the line through the two endpoints on the graph.