

Algebra 2 - Complex Conjugates and Complex Solutions - Lesson

Common Core N-CN.A.3, N-CN.C.7 · maths-pdf.fr/en/

The conjugate of $a + bi$ is $a - bi$, and a number times its conjugate equals a squared plus b squared. To divide complex numbers, multiply the numerator and denominator by the conjugate of the denominator. A quadratic equation with real coefficients and a negative discriminant has two complex solutions that are conjugates.

Algebra 2 · Common Core State Standards: **N-CN.A.3, N-CN.C.7**

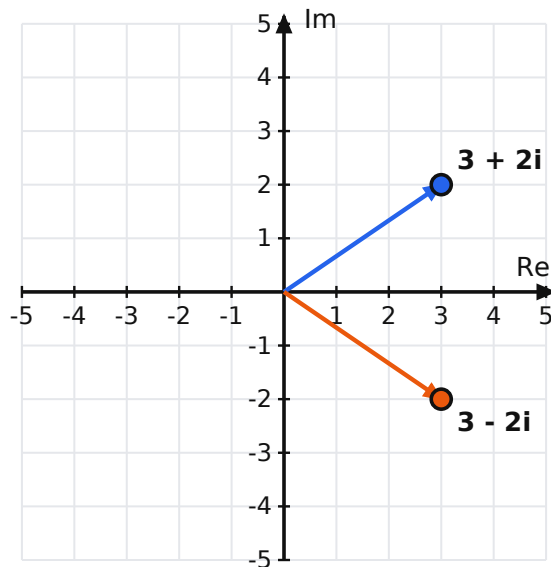
Key facts

- The conjugate of $a + bi$ is $a - bi$.
- z times its conjugate $= a^2 + b^2$, a real number.
- The modulus $|a + bi| = \sqrt{a^2 + b^2}$.
- Divide by multiplying top and bottom by the conjugate of the denominator.
- A negative discriminant gives a pair of complex conjugate solutions.

What is a complex conjugate?

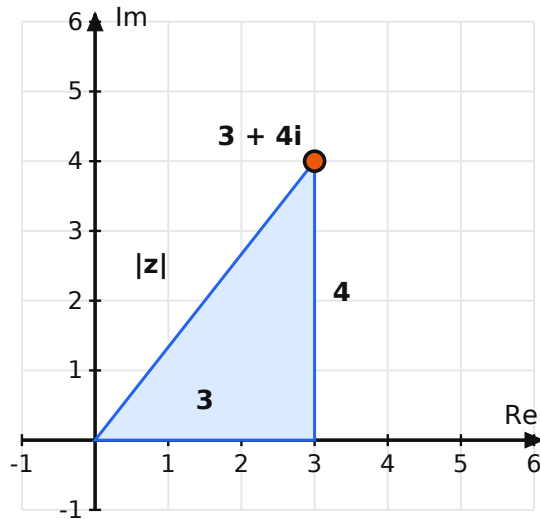
Complex conjugate and modulus

The **conjugate** of $z = a + bi$ is $\bar{z} = a - bi$: change the sign of the imaginary part. The **modulus** is $|z| = \sqrt{a^2 + b^2}$, the distance from the origin in the complex plane.



A conjugate pair is a reflection across the real axis.

A key fact: $z \cdot \bar{z} = (a+bi)(a-bi) = a^2 + b^2$. The product of a number and its conjugate is always a nonnegative real number. For $3+2i$, the product is $9+4=13$. Also, $z + \bar{z} = 2a$ is real.



The modulus $|z|$ is the length of the hypotenuse: the distance from the origin.

For $3+4i$, the modulus is $\sqrt{9+16}=5$.

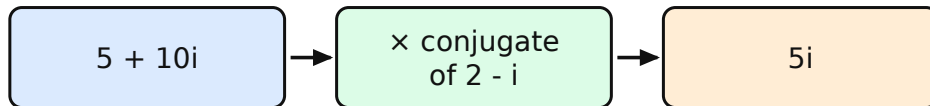
How do you divide complex numbers?

A fraction with i in the denominator is not in standard form. Multiply the numerator and denominator by the conjugate of the denominator. The new denominator becomes a real number.

1. Write the conjugate of the denominator.

2. Multiply the top and bottom by it.
3. Simplify using $i^2 = -1$.
4. Write the result as $a+bi$.

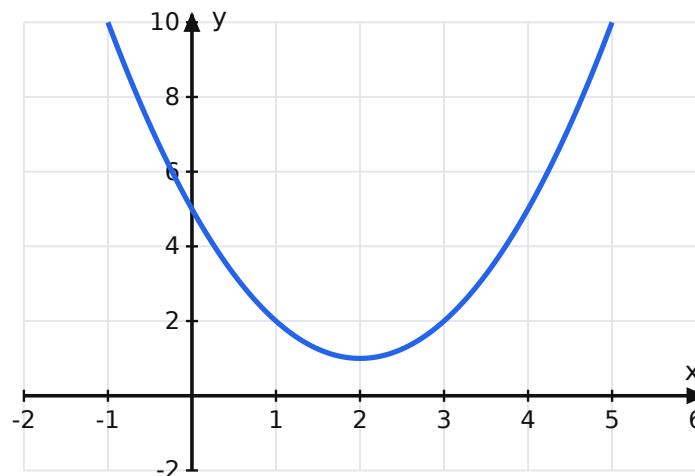
Example: $\frac{5+10i}{2-i} = \frac{(5+10i)(2+i)}{(2-i)(2+i)} = \frac{10+5i+20i+10i^2}{4+1} = \frac{25i}{5} = 5i$.



Multiply top and bottom by the conjugate of the denominator.

How do you solve a quadratic with complex solutions?

Use the quadratic formula. When the discriminant b^2-4ac is negative, the square root is imaginary, and the solutions are a conjugate pair $p \pm qi$. The graph of such a quadratic never touches the x-axis.



$y = x^2 - 4x + 5$ has no x-intercepts, so $x^2 - 4x + 5 = 0$ has no real solutions.

For $x^2-4x+5=0$: $x = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$.

Questions about complex conjugates and complex solutions

What is the conjugate of a complex number?

The conjugate of $a + bi$ is $a - bi$. Only the sign of the imaginary part changes. For example, the conjugate of $5 - 2i$ is $5 + 2i$.

Why do you multiply by the conjugate when dividing?

A number times its conjugate equals $a^2 + b^2$, which is real. That removes i from the denominator, so the quotient can be written in $a + bi$ form.

How do you find the modulus of a complex number?

Use the Pythagorean theorem: the modulus of $a + bi$ is the square root of $a^2 + b^2$. For $5 - 12i$, the modulus is the square root of 169, which is 13.

When does a quadratic equation have complex solutions?

When the discriminant $b^2 - 4ac$ is negative. Then the parabola does not cross the x -axis and the two solutions are complex conjugates.

Do complex solutions always come in pairs?

For a polynomial with real coefficients, yes. If $p + qi$ is a solution, then $p - qi$ is also a solution.