

Algebra 2 - Complex Conjugates and Complex Solutions - Answer Key

Common Core N-CN.A.3, N-CN.C.7 · maths-pdf.fr/en/

Every worksheet problem on complex conjugates, division and complex solutions is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where a missed sign in the conjugate can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **N-CN.A.3, N-CN.C.7**

Exercise 1 - Find a conjugate

Change the sign of the imaginary part: $4-3i$ becomes $4+3i$.

$$4+3i$$

Exercise 2 - Conjugate of an imaginary number

$6i = 0+6i$ has conjugate $0-6i = -6i$. The real number $-9 = -9+0i$ is its own conjugate.

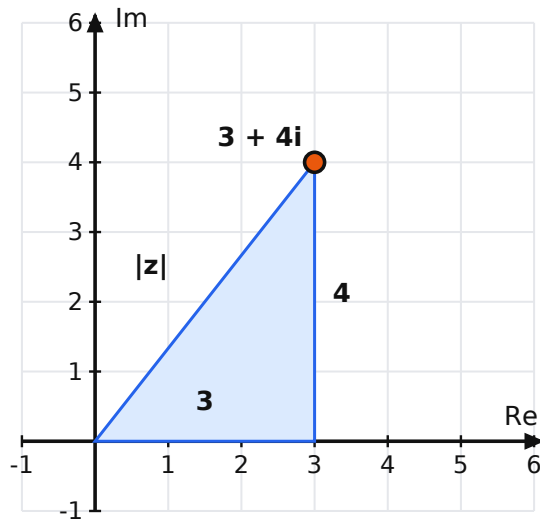
$$-6i \text{ and } -9$$

Exercise 3 - Product with the conjugate

$9-15i+15i-25i^2 = 9+25 = 34$. This equals $a^2+b^2 = 9+25$.

$$34$$

Exercise 4 - Modulus of $3 + 4i$



The modulus $|z|$ is the length of the hypotenuse: the distance from the origin.

$$|3+4i| = \sqrt{3^2+4^2} = \sqrt{25}.$$

5

Exercise 5 - Modulus with a radical

$$\sqrt{2^2+(-4)^2} = \sqrt{20} = \sqrt{4 \cdot 5}.$$

$2\sqrt{5}$

Exercise 6 - Divide a real number

Multiply the numerator and denominator by the conjugate of the denominator, $1-i$:

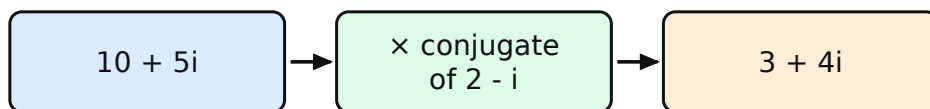
$$\frac{(6)(1-i)}{(1+i)(1-i)}$$

Numerator: $6-6i$. Denominator: $1+1=2$.

Divide each part by 2: $3-3i$.

$3-3i$

Exercise 7 - Divide complex numbers



Multiply top and bottom by the conjugate of the denominator.

Multiply the numerator and denominator by the conjugate of the denominator, $2+i$:

$$\frac{(10+5i)(2+i)}{(2-i)(2+i)}$$

Numerator: $15+20i$. Denominator: $4+1=5$.

Divide each part by 5: $3+4i$.

$$3+4i$$

Exercise 8 - Divide an imaginary number

Multiply the numerator and denominator by the conjugate of the denominator, $3-i$:

$$\frac{(5i)(3-i)}{(3+i)(3-i)}$$

Numerator: $5+15i$. Denominator: $9+1=10$.

Divide each part by 10: $\frac{1}{2}+\frac{3}{2}i$.

$$\frac{1}{2}+\frac{3}{2}i$$

Exercise 9 - Divide with fractions in the answer

Multiply the numerator and denominator by the conjugate of the denominator, $1+4i$:

$$\frac{(3+2i)(1+4i)}{(1-4i)(1+4i)}$$

Numerator: $-5+14i$. Denominator: $1+16=17$.

Divide each part by 17: $-\frac{5}{17}+\frac{14}{17}i$.

$$-\frac{5}{17}+\frac{14}{17}i$$

Exercise 10 - Divide by i

The conjugate of i is $-i$. Multiply top and bottom by $-i$: $\frac{(4+3i)(-i)}{i(-i)} = \frac{-4i-3i^2}{-i^2} = \frac{-4i+3}{1}$.

$$3-4i$$

Exercise 11 - Complex roots, easy

Identify $a=1, b=4, c=13$.

Discriminant: $b^2-4ac=(4)^2-4(1)(13)=-36$.

Since the discriminant is negative, the solutions are complex: $x = \frac{-4 \pm \sqrt{-36}}{2}$ with $\sqrt{-36}=6i$.

$$x = \frac{-4 \pm 6i}{2} = -2 \pm 3i.$$

$$-2+3i \text{ and } -2-3i$$

Exercise 12 - Complex roots, second

Identify $a=1, b=-6, c=10$.

Discriminant: $b^2-4ac=(-6)^2-4(1)(10)=-4$.

Since the discriminant is negative, the solutions are complex: $x = \frac{6 \pm \sqrt{-4}}{2}$ with $\sqrt{-4}=2i$.

$$x = \frac{6 \pm 2i}{2} = 3 \pm i.$$

$$3+i \text{ and } 3-i$$

Exercise 13 - Leading coefficient 2

Identify $a=2, b=2, c=5$.

Discriminant: $b^2-4ac=(2)^2-4(2)(5)=-36$.

Since the discriminant is negative, the solutions are complex: $x = \frac{-2 \pm \sqrt{-36}}{4}$ with $\sqrt{-36}=6i$.

$$x = \frac{-2 \pm 6i}{4} = -\frac{1}{2} \pm \frac{3}{2}i.$$

$$-\frac{1}{2}+\frac{3}{2}i \text{ and } -\frac{1}{2}-\frac{3}{2}i$$

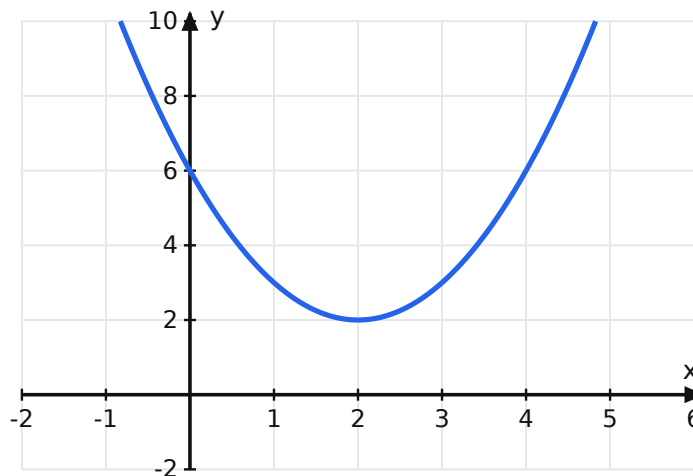
Exercise 14 - Pure imaginary roots

$$5x^2 = -20, \text{ so } x^2 = -4.$$

$$x = \pm\sqrt{-4} = \pm 2i.$$

$$x = 2i \text{ or } x = -2i$$

Exercise 15 - Use the graph



The vertex (2, 2) is above the x-axis.

The graph never touches the x-axis, so there are 0 real solutions.

Identify $a=1, b=-4, c=6$.

$$\text{Discriminant: } b^2 - 4ac = (-4)^2 - 4(1)(6) = -8.$$

Since the discriminant is negative, the solutions are complex: $x = \frac{4 \pm \sqrt{-8}}{2}$ with $\sqrt{-8} = 2\sqrt{2}i$.

$$\sqrt{-8} = 2i\sqrt{2}, \text{ so } x = \frac{4 \pm 2i\sqrt{2}}{2} = 2 \pm i\sqrt{2}.$$

No real solutions; $x = 2 \pm i\sqrt{2}$

Exercise 16 - Build an equation

Sum of roots: $(2+3i)+(2-3i)=4$. Product: $(2+3i)(2-3i)=4+9=13$.

A monic quadratic is $x^2 - (\text{sum})x + \text{product}$.

$$x^2 - 4x + 13 = 0$$

Exercise 17 - Ball that never gets there

$-16t^2 + 32t = 20$ becomes $16t^2 - 32t + 20 = 0$, which simplifies to $4t^2 - 8t + 5 = 0$.

Discriminant: $(-8)^2 - 4(4)(5) = 64 - 80 = -16$, which is negative.

The solutions are complex: $t = \frac{8 \pm 4i}{8} = 1 \pm \frac{1}{2}i$. No real time works, so the ball never reaches 20 feet. Its maximum height is $h(1) = 16$ feet.

Maya is wrong; the maximum is 16 ft

Exercise 18 - Impedance

Multiply the numerator and denominator by the conjugate of the denominator, $3 - i$:

$$\frac{(10 + 20i)(3 - i)}{(3 + i)(3 - i)}$$

Numerator: $50 + 50i$. Denominator: $9 + 1 = 10$.

Divide each part by 10: $5 + 5i$.

$5 + 5i$ ohms

Exercise 19 - Rationalize

Multiply by $\frac{1+2i}{1+2i}$: numerator $(1+2i)^2 = 1+4i-4 = -3+4i$, denominator $1+4=5$.

$$-\frac{3}{5} + \frac{4}{5}i$$

Exercise 20 - Iterate

$$z_0^2 = (1+i)^2 = 1+2i-1 = 2i.$$

$$z_1 = 2i + (-1+2i) = -1+4i.$$

$$|z_1| = \sqrt{1+16} = \sqrt{17}.$$

$$z_1 = -1+4i, |z_1| = \sqrt{17}$$

Exercise 21 - Second root

Complex solutions of a real quadratic come in conjugate pairs, so the other solution is $3+2i$.

Sum $=6$, product $=9+4=13$, so $b=-6$ and $c=13$.

$$3+2i; x^2-6x+13=0$$