

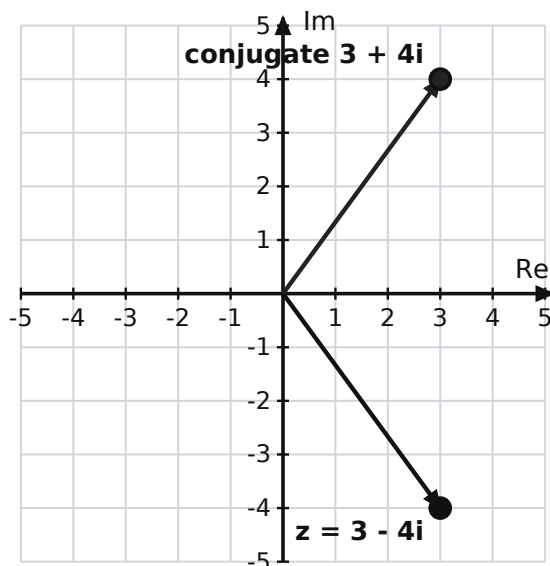
Algebra 2 - Complex Conjugates and Complex Solutions - Test Answer Key

Common Core N-CN.A.3, N-CN.C.7 · maths-pdf.fr/en/

Use this answer key and scoring guide to grade the test on complex conjugates, division and complex solutions (N-CN.A.3). Each solution shows the full reasoning, and the scoring note explains how many points to award for correct setup, correct work, and a correct final answer. Partial credit rewards a sound method even when a small arithmetic slip changes the result.

Algebra 2 · Common Core State Standards: **N-CN.A.3, N-CN.C.7**

Question 1 - Conjugate and modulus [2 pts]



z and its conjugate.

$$z = 3 - 4i, \text{ conjugate } 3 + 4i, |z| = \sqrt{9 + 16} = 5.$$

$$3 - 4i; 3 + 4i; 5$$

Scoring: 1 point for z and the conjugate, 1 point for the modulus.

Question 2 - Divide [3 pts]

- a. Multiply the numerator and denominator by the conjugate of the denominator, $1 + i$:

$$\frac{(7+i)(1+i)}{(1-i)(1+i)}$$

Numerator: $6+8i$. Denominator: $1+1=2$.

Divide each part by 2: $3+4i$.

$$3+4i$$

b. Multiply the numerator and denominator by the conjugate of the denominator, $-i$:

$$\frac{(2-3i)(-i)}{(i)(-i)}$$

Numerator: $-3-2i$. Denominator: $0+1=1$.

Divide each part by 1: $-3-2i$.

$$-3-2i$$

Scoring: Part (a): 1 point for multiplying by the conjugate, 1 point for the answer. Part (b): 1 point.

Question 3 - Quadratic formula [3 pts]

Identify $a=1, b=-2, c=5$.

Discriminant: $b^2-4ac=(-2)^2-4(1)(5)=-16$.

Since the discriminant is negative, the solutions are complex: $x=\frac{2\pm\sqrt{-16}}{2}$ with $\sqrt{-16}=4i$.

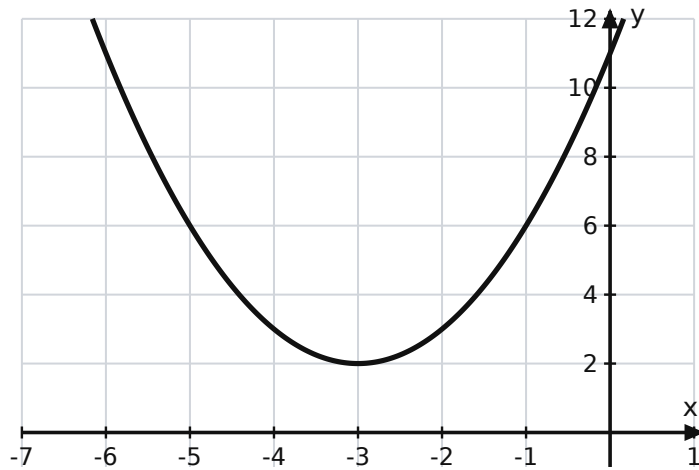
$$x=\frac{2\pm 4i}{2}=1\pm 2i.$$

Second equation: $x^2=-16$, so $x=\pm 4i$.

$$1\pm 2i; \pm 4i$$

Scoring: First equation: 1 point for the discriminant, 1 point for both solutions. Second equation: 1 point.

Question 4 - Count real solutions [2 pts]



Vertex $(-3, 2)$: no x-intercepts.

The vertex $(-3, 2)$ is above the x-axis and the parabola opens up, so there are 0 real solutions.

$b^2 - 4ac = 36 - 44 = -8 < 0$, which confirms two complex solutions.

0 real solutions; discriminant -8

Scoring: 1 point for reading the graph, 1 point for the discriminant.

Question 5 - Write the equation [2 pts]

Sum = 2. Product = $1 + 16 = 17$.

$$x^2 - 2x + 17 = 0$$

Scoring: 1 point for the sum and product, 1 point for the equation.

Question 6 - Impedance [2 pts]

Multiply the numerator and denominator by the conjugate of the denominator, $2 - i$:

$$\frac{(12 + 16i)(2 - i)}{(2 + i)(2 - i)}$$

Numerator: $40 + 20i$. Denominator: $4 + 1 = 5$.

Divide each part by 5: $8 + 4i$.

$8 + 4i$ ohms

Scoring: 1 point for the conjugate step, 1 point for the answer.

Question 7 - Modulus of a product [2 pts]

$$(1+2i)(1-2i)=1+4=5. \quad |z|^2=(\sqrt{5})^2=5.$$

Both equal 5

Scoring: 1 point for the product, 1 point for the comparison.

Question 8 - Projectile [2 pts]

$$-16t^2+48t-40=0 \text{ simplifies to } 2t^2-6t+5=0. \text{ Discriminant: } 36-40=-4.$$

No: the discriminant is negative (max height 36 ft)

Scoring: 1 point for the equation, 1 point for the conclusion.

Question 9 - Solve with a leading coefficient [2 pts]

Identify $a=3, b=-6, c=7$.

$$\text{Discriminant: } b^2-4ac=(-6)^2-4(3)(7)=-48.$$

Since the discriminant is negative, the solutions are complex: $x=\frac{6\pm\sqrt{-48}}{6}$ with $\sqrt{-48}=4\sqrt{3}i$.

$$x=\frac{6\pm 4i\sqrt{3}}{6} \text{ reduces to } 1\pm\frac{2\sqrt{3}}{3}i.$$

$$x=1\pm\frac{2\sqrt{3}}{3}i$$

Scoring: 1 point for the discriminant, 1 point for the simplified solutions.

Total
20 points