

Algebra 2 - Complex Numbers: Adding, Subtracting and Multiplying - Lesson

Common Core N-CN.A.1, N-CN.A.2 · maths-pdf.fr/en/

A complex number has the form $a + bi$, where a and b are real and i is the imaginary unit with i squared equal to -1 . Add and subtract by combining real parts and imaginary parts separately. Multiply with the distributive property, then replace each i squared with -1 .

Algebra 2 · Common Core State Standards: **N-CN.A.1, N-CN.A.2**

Key facts

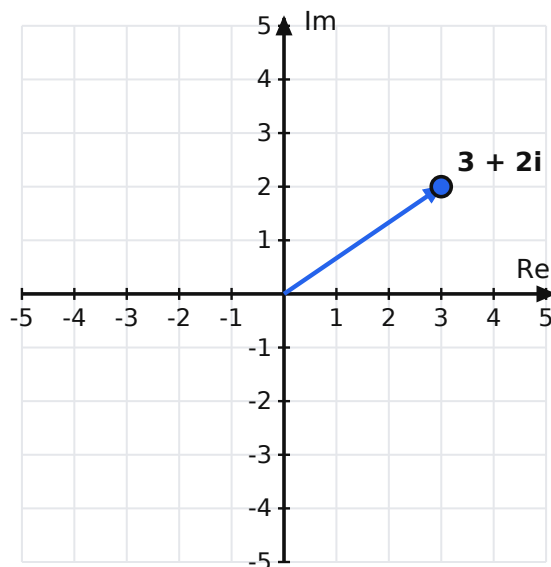
- The imaginary unit i satisfies $i^2 = -1$, so $\sqrt{(-n)} = i\sqrt{n}$ for $n > 0$.
- A complex number $a + bi$ has real part a and imaginary part b .
- Add or subtract real parts and imaginary parts separately.
- Multiply by distributing, then replace i^2 with -1 .
- Powers of i repeat: $i, -1, -i, 1$.

What is the imaginary unit i ?

No real number squares to a negative, so mathematicians define a new number that does. This number lets us take square roots of negative numbers and solve equations like $x^2 + 1 = 0$.

Imaginary unit and complex numbers

The **imaginary unit** is i , defined by $i^2 = -1$, so $i = \sqrt{-1}$. A **complex number** has the form $a + bi$, where a is the **real part** and b is the **imaginary part**. For a positive number n , $\sqrt{-n} = i\sqrt{n}$.

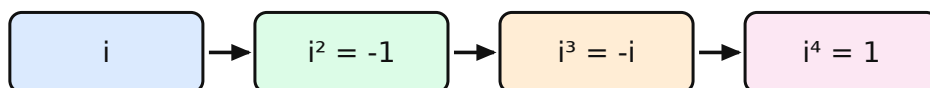


The complex number $3 + 2i$ is the point (3, 2): real part on the horizontal axis, imaginary part on the vertical axis.

Every real number is also complex, because $5 = 5 + 0i$. The number $-4i$ has real part 0 and imaginary part -4 . For example, $\sqrt{-36} = 6i$ and $\sqrt{-20} = 2i\sqrt{5}$.

How do the powers of i behave?

Start with $i^1 = i$ and $i^2 = -1$. Then $i^3 = i^2 \cdot i = -i$ and $i^4 = (i^2)^2 = 1$. After that the pattern repeats every four powers.

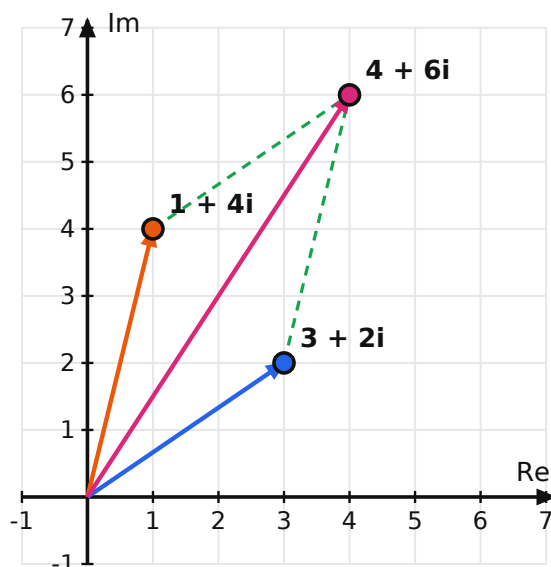


The powers of i repeat in a cycle of length 4.

To find i^n , divide n by 4 and use the remainder. For i^{27} , $27 = 4 \cdot 6 + 3$, so $i^{27} = i^3 = -i$.

How do you add and subtract complex numbers?

Add or subtract the real parts and the imaginary parts separately, just like combining like terms. For $(3+2i)+(1+4i)$ the real parts give $3+1=4$ and the imaginary parts give $2+4=6$, so the sum is $4+6i$. On the plane, the sum is the far corner of the parallelogram built from the two numbers.



Adding complex numbers is vector addition: $3 + 2i$ and $1 + 4i$ give $4 + 6i$.

How do you multiply complex numbers?

Use the distributive property, then replace i^2 with -1 . Follow these steps for $(2+3i)(4-i)$.

1. Multiply each term in the first factor by each term in the second.
2. Replace i^2 with -1 .
3. Combine the real parts and the imaginary parts.

Example: $(2+3i)(4-i) = 8 - 2i + 12i - 3i^2 = 8 + 10i + 3 = 11 + 10i$.

	4	-1i
2	8	-2i
3i	12i	$-3i^2 = 3$

The four partial products 8, -2i, 12i and $-3i^2$ add to $11 + 10i$.

The set of complex numbers is closed under addition, subtraction, and multiplication, and it keeps the commutative, associative, and distributive properties.

Questions about complex numbers: adding, subtracting and multiplying

What is i in math?

The letter i stands for the imaginary unit, defined by i squared equals negative 1. It lets you write square roots of negative numbers, such as the square root of -9 equals $3i$.

What is the difference between real and imaginary numbers?

A real number is a point on the ordinary number line. An imaginary number has the form bi with b real and b not zero. A complex number $a + bi$ combines a real part and an imaginary part.

How do you simplify powers of i?

Divide the exponent by 4 and keep the remainder. A remainder of 0, 1, 2, or 3 gives 1, i , -1 , or $-i$. For example, i to the 50th has remainder 2, so it equals -1 .

Why does i squared equal -1 when multiplying complex numbers?

Because i squared is defined as -1 . After you distribute, every i -squared term becomes a real number, and the final answer is written in $a + bi$ form.

Can a complex number be graphed?

Yes. In the complex plane the horizontal axis shows the real part and the vertical axis shows the imaginary part, so $a + bi$ is plotted at the point (a, b) .