

Algebra 2 - Composition and Inverse Functions - Lesson

Common Core F-BF.A.1c, F-BF.B.4a · maths-pdf.fr/en/

The composition $f(g(x))$ applies g first and then f . The inverse of a function undoes it: swap x and y and solve for y , then check that $f(f^{-1}(x)) = x$. The graph of the inverse is the reflection of the graph across the line $y = x$. Functions that are not one-to-one need a restricted domain.

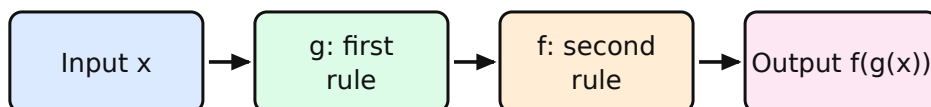
Algebra 2 · Common Core State Standards: **F-BF.A.1c, F-BF.B.4a**

Key facts

- $f(g(x))$: apply g first, then f .
- Composition order matters.
- Inverse: swap x and y , then solve for y .
- $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
- Graphs of f and f^{-1} are mirror images over $y = x$.

What is the composition of two functions?

To **compose** functions, use the output of one as the input of the other. The notation $f(g(x))$ means: apply g first, then f . For $f(x) = 2x + 3$ and $g(x) = x^2$, $f(g(x)) = 2x^2 + 3$ but $g(f(x)) = (2x + 3)^2$. Order matters.



Composition: g acts first, then f acts on the result.

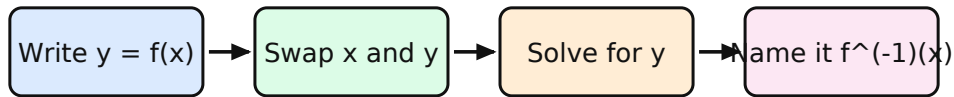
The domain of $f(g(x))$ includes only inputs that are in the domain of g and whose outputs are in the domain of f . For $f(x) = \frac{1}{x-2}$ and $g(x) = x+3$, $f(g(x)) = \frac{1}{x+1}$, so $x \neq -1$.

What is an inverse function?

Inverse function

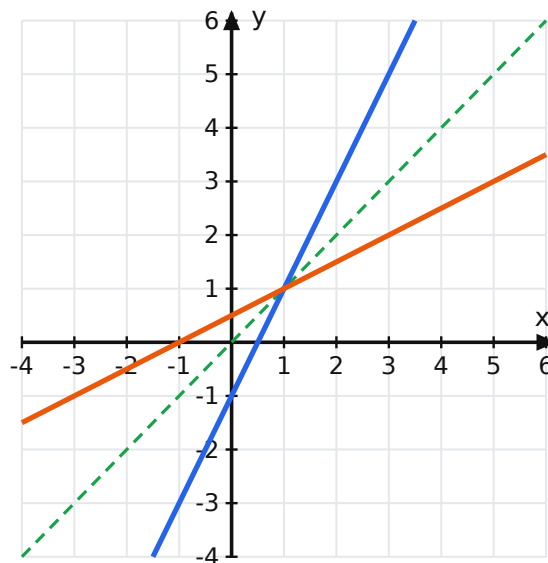
The **inverse** of f is the function f^{-1} that undoes f : $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$. If

(a,b) is on the graph of f , then (b,a) is on the graph of f^{-1} . A function has an inverse only if it is one-to-one (it passes the horizontal line test).



Finding an inverse in four steps.

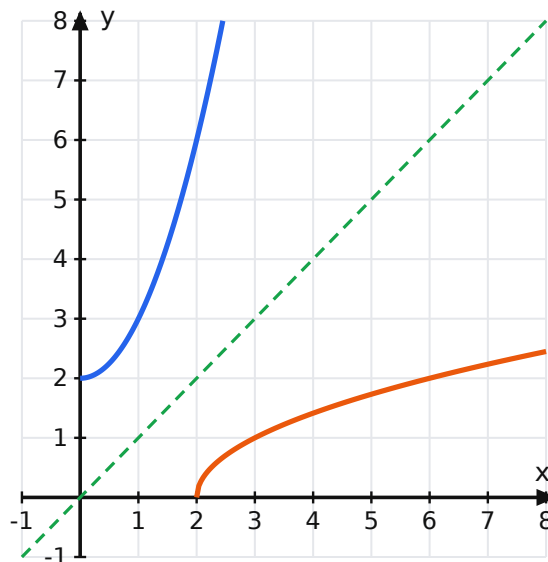
Example: $f(x) = 3x - 6$. Write $y = 3x - 6$, swap to $x = 3y - 6$, and solve: $y = \frac{x+6}{3}$. So $f^{-1}(x) = \frac{x+6}{3}$.
Check: $f\left(\frac{x+6}{3}\right) = (x+6) - 6 = x$.



The graph of f^{-1} is the reflection of f across the line $y = x$ (dashed).

How do you find an inverse when the function is not one-to-one?

The function $f(x) = x^2$ fails the horizontal line test, because $f(2) = f(-2)$. Restrict the domain to $x \geq 0$. Then f is one-to-one and $f^{-1}(x) = \sqrt{x}$. The restriction decides which root to use: for $x \leq 0$ the inverse would be $-\sqrt{x}$.



$$f(x) = x^2 + 2 \text{ with } x \geq 0 \text{ and } f^{-1}(x) = \sqrt{x - 2}.$$

You can also find inverses from tables by swapping the columns, and check any pair by composing both ways. Real-world inverses answer the reverse question: if $F = \frac{9}{5}C + 32$ converts Celsius to Fahrenheit, the inverse $C = \frac{5}{9}(F - 32)$ converts back.

x	f(x)		x	f ⁻¹ (x)
0	3		3	0
1	5		5	1
2	7		7	2

An inverse swaps inputs and outputs.

Questions about composition and inverse functions

What does $f(g(x))$ mean?

It means apply g to x first, then apply f to the result. The inner function g works first.

How do you find the inverse of a function?

Write $y = f(x)$, swap x and y , solve for y , and call the result f inverse of x . Then verify that composing in both orders gives x .

How can you tell if two functions are inverses?

Compose them in both orders. If $f(g(x)) = x$ and $g(f(x)) = x$ for all x in the domains, they are inverses.

What does the graph of an inverse look like?

The graph of f inverse is the reflection of the graph of f across the line $y = x$, because every point (a, b) becomes (b, a) .

Why do some functions need a restricted domain to have an inverse?

A function must be one-to-one to have an inverse. For x squared, two inputs give the same output, so we restrict the domain to x greater than or equal to 0.