

Algebra 2 - Composition and Inverse Functions - Answer Key

Common Core F-BF.A.1c, F-BF.B.4a · maths-pdf.fr/en/

Every worksheet problem on composition and inverse functions is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where composing in the wrong order or forgetting to swap x and y can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **F-BF.A.1c, F-BF.B.4a**

Exercise 1 - Evaluate a composition

$g(2) = 4$, so $f(g(2)) = f(4) = 11$. $f(2) = 7$, so $g(f(2)) = g(7) = 49$.

$f(g(2)) = 11$ and $g(f(2)) = 49$

Exercise 2 - Compose expressions

$f(g(x)) = f(3x+4)$.

$g(f(x)) = g(x+4) = 3(x+4) = 3x+12$.

$3x + 4$ and $3x + 12$; not equal

Exercise 3 - Compose from tables

x	1	2	3
$g(x)$	4	1	2
$f(g(x))$	7	5	0

$f(g(x))$ for $x = 1, 2, 3$.

$g(1) = 4$, $f(4) = 7$. $g(2) = 1$, $f(1) = 5$. $g(3) = 2$, $f(2) = 0$.

7, 5, 0

Exercise 4 - Composition with a radical

$$f(g(x)) = \sqrt{x^2+1}-1 = \sqrt{x^2}.$$

$\sqrt{x^2} = |x|$. The domain is all real numbers, because $x^2+1 \geq 1$.

$$f(g(x)) = |x|$$

Exercise 5 - Domain of a composition

$$f(g(x)) = \frac{1}{(x+3)-2} = \frac{1}{x+1}.$$

The denominator is 0 when $x = -1$.

$$f(g(x)) = \frac{1}{x+1}, \quad x \neq -1$$

Exercise 6 - Inverse from a table

The outputs rise by 3 each step: slope 3. $f(1) = 7$ gives $f(x) = 3x + 4$.

Swap: $x = 3y + 4$, so $y = \frac{x-4}{3}$.

$$f^{-1}(25) = \frac{25-4}{3} = 7.$$

$$f(x) = 3x + 4; \quad f^{-1}(x) = \frac{x-4}{3}; \quad \text{input 7}$$

Exercise 7 - Inverse with a fraction

Swap: $x = \frac{y+5}{2}$. Solve: $y = 2x - 5$.

Check: $f(2x-5) = \frac{2x-5+5}{2} = x$.

$$f^{-1}(x) = 2x - 5$$

Exercise 8 - Inverse of a cube

Swap: $x = y^3 + 1$, so $y^3 = x - 1$.

$$f^{-1}(x) = \sqrt[3]{x-1}$$

Exercise 9 - Inverse of a rational function

Swap: $x = \frac{2}{y-1} + 3$. Subtract 3: $x-3 = \frac{2}{y-1}$.

So $y-1 = \frac{2}{x-3}$ and $y = \frac{2}{x-3} + 1$.

$$f^{-1}(x) = \frac{2}{x-3} + 1$$

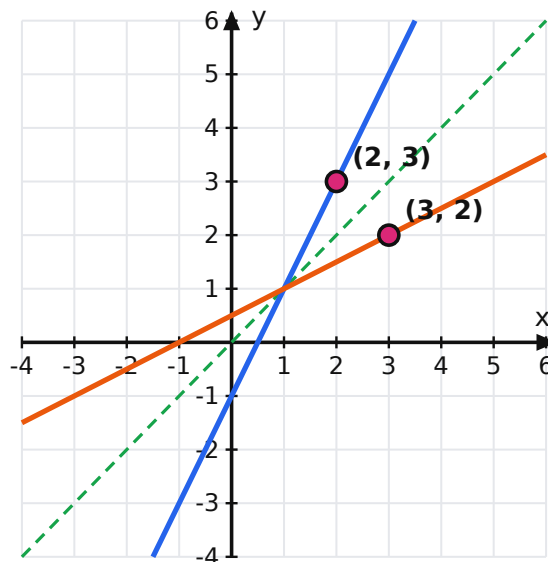
Exercise 10 - Verify inverses

$$f(g(x)) = 4 \cdot \frac{x+8}{4} - 8 = x + 8 - 8 = x.$$

$$g(f(x)) = \frac{4x-8+8}{4} = x.$$

Both compositions equal x , so they are inverses

Exercise 11 - Read the graph



$(2, 3)$ on f gives $(3, 2)$ on f^{-1} .

$f(2) = 3$, so $(2, 3)$ is on f and $(3, 2)$ is on f^{-1} .

Algebra: $f^{-1}(x) = \frac{x+1}{2}$, so $f^{-1}(3) = 2$.

$$f^{-1}(3) = 2$$

Exercise 12 - Inverse of a table

x	5	3	2	-1
$f^{-1}(x)$	-2	0	1	4

Swap the rows.

Swap inputs and outputs. The row of outputs of f becomes the inputs of the inverse.

$$f^{-1}(2) = 1$$

Exercise 13 - Restricted quadratic

Swap: $x = y^2 + 2$, so $y^2 = x - 2$. Since the original domain is $x \geq 0$, the inverse's outputs must be nonnegative: $y = \sqrt{x - 2}$.

The domain of the inverse equals the range of f : $x \geq 2$.

$$f^{-1}(x) = \sqrt{x - 2}, \text{ domain } x \geq 2$$

Exercise 14 - Why no inverse?

$f(2) = 4$ and $f(-2) = 4$. A horizontal line at $y = 4$ meets the graph twice, so f is not one-to-one.

An inverse would have to send 4 to both 2 and -2, which a function cannot do.

Not one-to-one; restrict the domain to $x \geq 0$ (or $x \leq 0$)

Exercise 15 - Vertex form with a restriction

Swap: $x = (y - 3)^2 - 1$, so $(y - 3)^2 = x + 1$.

Because $y \geq 3$, take the positive root: $y - 3 = \sqrt{x + 1}$.

$$f^{-1}(x) = \sqrt{x + 1} + 3, \text{ domain } x \geq -1$$

Exercise 16 - Temperature

Solve for C : $F - 32 = \frac{9}{5}C$, so $C = \frac{5}{9}(F - 32)$.

$$C = \frac{5}{9}(86 - 32) = \frac{5}{9}(54) = 30.$$

$$C = \frac{5}{9}(F - 32); \mathbf{86^{\circ}F = 30^{\circ}C}$$

Exercise 17 - Taxi fare

Swap: $m = 3 + 2.5c$, so $c = \frac{m-3}{2.5}$. In words: miles = $\frac{\text{fare}-3}{2.5}$.

For \$18: $\frac{18-3}{2.5} = 6$.

6 miles

Exercise 18 - Order matters

Discount first: $f(60) = 48$, then $g(48) = 43$.

Coupon first: $g(60) = 55$, then $f(55) = 44$.

Discount first: \$43 beats \$44

Exercise 19 - Growing circle

$$A(r(t)) = \pi(3t)^2 = 9\pi t^2.$$

$$A(2) = 9\pi \cdot 4 = 36\pi \approx 113.1.$$

$$A(t) = 9\pi t^2; \mathbf{\text{about } 113.1 \text{ square inches}}$$

Exercise 20 - Find the error

$f(3) = 7$, so an inverse must send 7 back to 3. His function gives $\frac{1}{15}$.

Correct: $x = 2y + 1$, so $y = \frac{x-1}{2}$; $\frac{7-1}{2} = 3$.

$$f^{-1}(x) = \frac{x-1}{2}$$

Exercise 21 - Find the outer function

Let $u = x + 2$, so $x = u - 2$. Then $f(u) = (u - 2)^2 + 4(u - 2) + 7 = u^2 - 4u + 4 + 4u - 8 + 7 = u^2 + 3$.

Check: $(x + 2)^2 + 3 = x^2 + 4x + 7$.

$$f(x) = x^2 + 3$$