

Algebra 2 - Exponential Growth and Decay - Lesson

Common Core F-LE.A.2, F-LE.B.5, A-SSE.B.3c, F-IF.C.8b · maths-pdf.fr/en/

An exponential function $f(x) = a \cdot b^x$ multiplies by the factor b for each step in x . It grows when b is greater than 1 and decays when b is between 0 and 1, and a percent rate r gives $b = 1 + r$ or $1 - r$. Compound interest and continuous growth are exponential models.

Algebra 2 · Common Core State Standards: **F-LE.A.2, F-LE.B.5, A-SSE.B.3c, F-IF.C.8b**

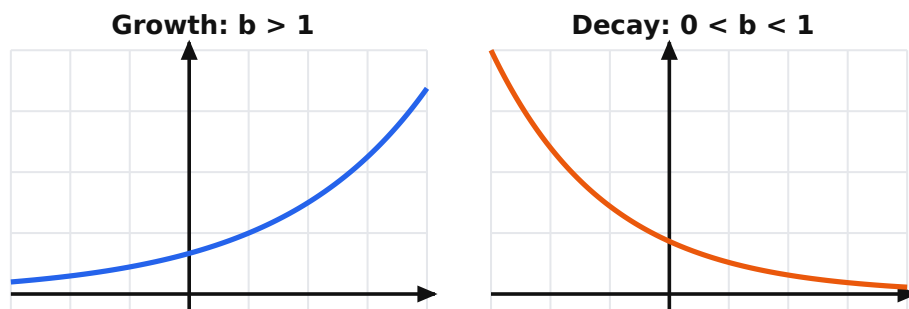
Key facts

- $f(x) = a \cdot b^x$: a is the initial value, b the growth factor.
- Growth factor $b = 1 + r$ (growth) or $1 - r$ (decay).
- Equal steps in x multiply the output by b .
- Compound interest: $A = P(1 + r/n)^{nt}$; continuous: $A = Pe^{rt}$.
- Rewrite b^x expressions to find equivalent rates.

What is an exponential function?

Exponential function

An **exponential function** has the form $f(x) = a \cdot b^x$ with $a \neq 0$, $b > 0$, and $b \neq 1$. The constant a is the initial value (the y-intercept) and b is the **growth factor** per unit of x . If $b > 1$ the function grows; if $0 < b < 1$ it decays.



Growth (left) and decay (right) both have a horizontal asymptote at $y = 0$.

In a linear function equal steps in x add the same amount. In an exponential function equal steps in x multiply by the same factor. A table with outputs 5,10,20,40 for $x=0,1,2,3$ shows ratio 2 each time, so $f(x) = 5 \cdot 2^x$.

How do percent rates turn into growth factors?

A quantity that changes by a percent r each period has growth factor $b = 1 + r$ for growth and $b = 1 - r$ for decay. A town of 12,000 people growing 3.5% a year follows $P = 12,000(1.035)^t$. A car worth \$24,000 that loses 15% each year follows $V = 24,000(0.85)^t$.

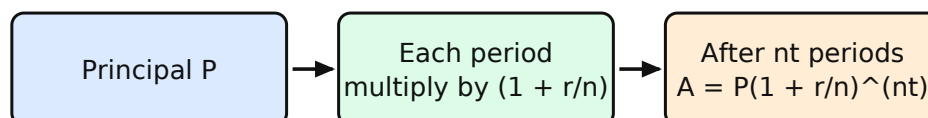
Situation	Rate	Factor b
Growth 3.5% per year	+0.035	1.035
Decay 15% per year	-0.15	0.85
Doubles every period	+100%	2
Half-life	-50%	0.5

Growth factor = 1 + rate.

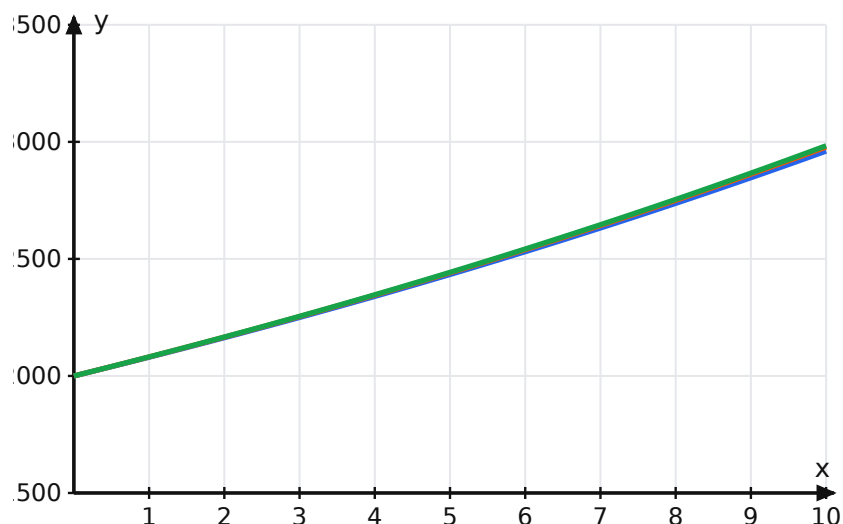
Exponential models are used whenever change is proportional to the current amount: bank balances, populations, medicine in the body, and the value of a car. To build a model, identify the starting value a and the factor b per time unit. To interpret a model, read a as the amount at time 0 and convert b back to a percent rate with $r = b - 1$. Always match the time units: if t is in years, the factor is per year.

How does compound interest work?

When interest is added n times a year at annual rate r , the balance after t years is $A = P(1 + \frac{r}{n})^{nt}$. With continuous compounding, $A = Pe^{rt}$, where $e \approx 2.71828$. For \$2,000 at 4% compounded quarterly for 5 years, $A = 2000(1.01)^{20} \approx 2,440.38$.



Compound interest repeats a multiplication.



Annual, quarterly, and continuous compounding of \$2,000 at 4%: more frequent compounding gives slightly more.

Exponential expressions can be rewritten to reveal other rates. For example $1.02^{12t} = (1.02^{12})^t \approx 1.268^t$: a 2% monthly rate is about 26.8% per year, not 24%. And $2^{3x} = (2^3)^x = 8^x$.

Questions about exponential growth and decay

What is the growth factor of an exponential function?

The growth factor b is the number you multiply by for each one-unit step in x . In $f(x) = a$ times b to the x , b is greater than 1 for growth and between 0 and 1 for decay.

How do you find a percent growth factor?

Add the rate to 1 for growth, or subtract it from 1 for decay. A 6% increase has $b = 1.06$ and a 6% decrease has $b = 0.94$.

What is the compound interest formula?

$A = P(1 + r/n)^{nt}$, where P is the principal, r the annual rate, n the number of compoundings per year, and t the time in years.

What is the number e ?

The number e is about 2.71828. It is used for continuous growth, $A = Pe^{rt}$, and it arises as the limit of compounding more and more often.

How is an exponential function different from a linear function?

A linear function adds the same amount each step. An exponential function multiplies by the

same factor each step, so it eventually grows faster than any linear function.