

Algebra 2 - Logarithms - Lesson

Common Core F-LE.A.4, F-BF.B.5, F-IF.C.7e · maths-pdf.fr/en/

A logarithm is an exponent: log base b of x equals y means b to the power y equals x. Logarithms are inverses of exponential functions, so their graphs are reflections across $y = x$, and they turn products into sums. Use logarithms and their properties to solve exponential equations such as 2 to the x equals 20.

Algebra 2 · Common Core State Standards: **F-LE.A.4, F-BF.B.5, F-IF.C.7e**

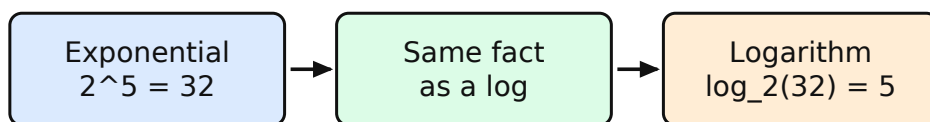
Key facts

- $\log_b(x) = y$ means $b^y = x$.
- $\log_b(1) = 0$ and $\log_b(b) = 1$.
- Product, quotient, and power rules: $\log(xy) = \log x + \log y$, etc.
- Change of base: $\log_b(x) = \ln x / \ln b$.
- The graph of $\log_b(x)$ is the reflection of b^x over $y = x$, with asymptote $x = 0$.

What is a logarithm?

Logarithm

The **logarithm base b** of x is the exponent that b must be raised to in order to get x :
 $\log_b(x) = y$ means $b^y = x$, for $b > 0$, $b \neq 1$, and $x > 0$.



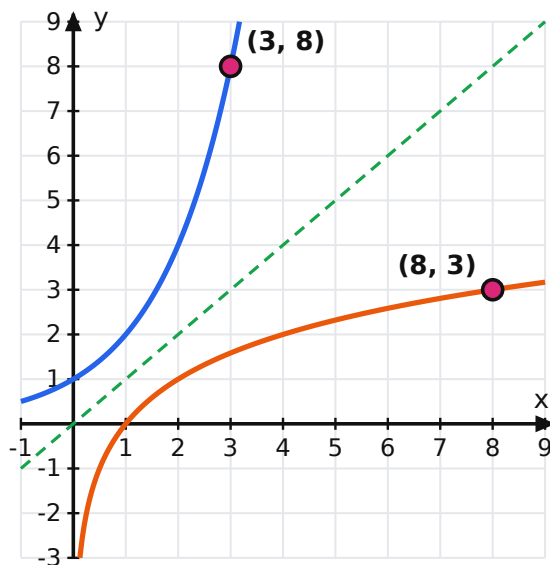
A logarithm is an exponent: $\log_b(x) = y$ means $b^y = x$.

So $\log_2(32) = 5$ because $2^5 = 32$, and $\log_{10}(1000) = 3$. Two bases have special names. The **common logarithm** $\log x$ has base 10, and the **natural logarithm** $\ln x$ has base $e \approx 2.718$. Because $b^0 = 1$ and $b^1 = b$, every logarithm satisfies $\log_b(1) = 0$ and $\log_b(b) = 1$.

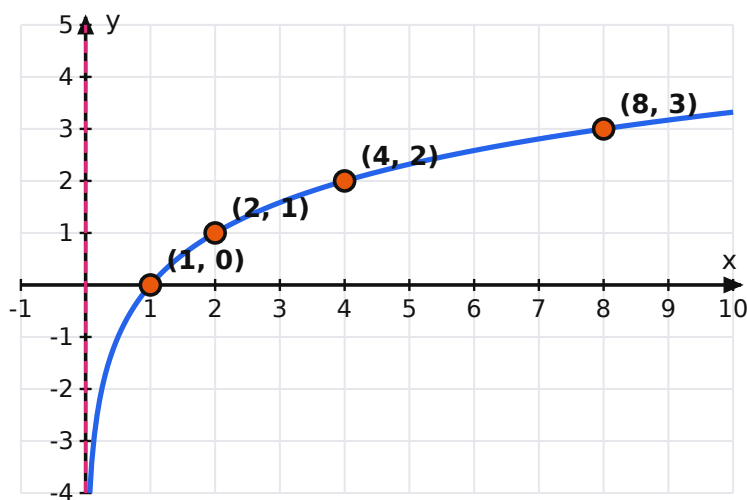
How is the graph of a logarithm related to an exponential?

The function $f(x) = \log_b(x)$ is the inverse of $g(x) = b^x$. Swapping x and y reflects the graph over the line $y = x$. The domain of the logarithm is $x > 0$, the range is all real numbers, and the y -

axis is a vertical asymptote.



$y = 2^x$ (blue) and $y = \log_2 x$ (orange) are reflections across $y = x$.



$y = \log_2 x$ passes through $(1, 0)$, $(2, 1)$, $(4, 2)$, $(8, 3)$.

What are the properties of logarithms?

Because logarithms are exponents, they turn multiplication into addition. These rules hold for positive x, y .

Property	Rule	Example
Product	$\log_b(xy) = \log_b(x) + \log_b(y)$	$\log_2(8 \cdot 4) = 3 + 2$
Quotient	$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$	$\log_3\left(\frac{81}{9}\right) = 4 - 2$
Power	$\log_b(x^n) = n \log_b(x)$	$\log_5(25^3) = 3 \cdot 2$

Property	Rule	Example
Change of base	$\log_b(x) = \frac{\ln x}{\ln b}$	$\log_2(20) = \frac{\ln 20}{\ln 2} \approx 4.32$

How do you solve an exponential equation?

Isolate the exponential and take a logarithm of both sides. For $2^x = 20$, $x = \log_2(20) = \frac{\ln 20}{\ln 2} \approx 4.32$. With a coefficient in the exponent, use the power rule: $5^{2x} = 100$ gives $2x \ln 5 = \ln 100$, so $x = \frac{\ln 100}{2 \ln 5} \approx 1.43$. Always check logarithmic equations for extraneous solutions, because the argument of a log must be positive.

x	1/8	1/2	1	2	8
log ₂ x	-3	-1	0	1	3

Values of log base 2 of x.

Questions about logarithms

What is a logarithm?

A logarithm is an exponent. The log base b of x is the power you raise b to in order to get x, so log base 2 of 32 is 5 because 2 to the 5th is 32.

What is the difference between log and ln?

The common log, log, has base 10, and the natural log, ln, has base e, about 2.718. Both follow the same properties.

What are the properties of logarithms?

The log of a product is the sum of logs, the log of a quotient is the difference, and the log of a power brings the exponent down in front.

How do you solve an equation like 3 to the x equals 50?

Take a logarithm of both sides and use the power rule: x times ln 3 equals ln 50, so x equals ln 50 divided by ln 3, about 3.56.

Why can't you take the log of a negative number?

No real power of a positive base gives a negative number, so the logarithm of a negative number or zero is undefined in the real numbers.