

Algebra 2 - Systems of Linear and Quadratic Equations - Lesson

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To solve a system of a linear and a quadratic equation, substitute the linear expression for y into the quadratic, solve for x , then find y for each x . The line meets the parabola in two points, one point, or none, depending on the sign of the discriminant.

Algebra 2 · Common Core State Standards: **A-REI.C.7**

Key facts

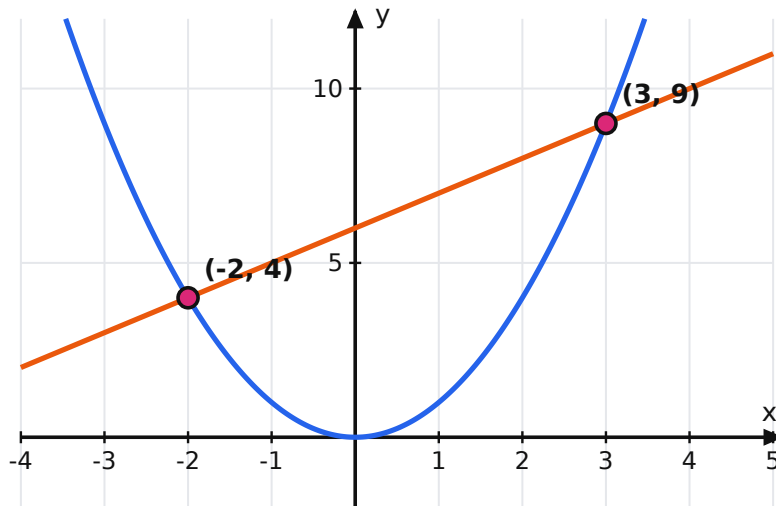
- Substitute the line into the quadratic to get one equation in x .
- Solve the resulting quadratic; then find y for each x .
- $D > 0$: two intersection points; $D = 0$: one (tangent); $D < 0$: none.
- A solution is an ordered pair (x, y) .
- The method also works for a line and a circle.

How do you solve a system with a line and a parabola?

A system of one linear and one quadratic equation asks where a line and a parabola meet. Use substitution: replace y in the quadratic with the linear expression, solve the resulting quadratic for x , then find each y .

1. Solve the linear equation for y if it is not already.
2. Substitute into the quadratic equation to get an equation in x only.
3. Write it as $ax^2+bx+c=0$ and solve by factoring or the formula.
4. Substitute each x into the linear equation to get y .

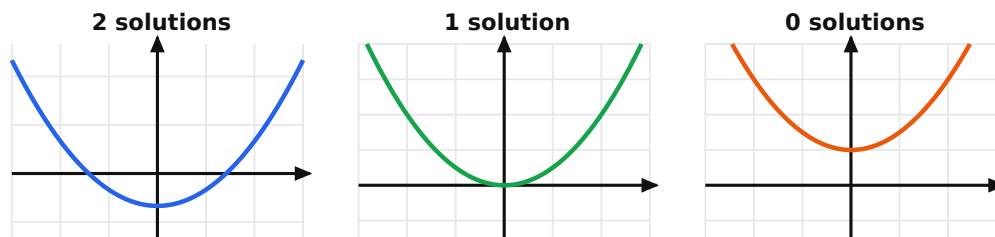
Example: $y = x^2$ and $y = x+6$. Then $x^2 = x+6$, so $x^2 - x - 6 = 0$ and $(x-3)(x+2) = 0$. The solutions are $(3,9)$ and $(-2,4)$.



The line and parabola meet at $(-2, 4)$ and $(3, 9)$.

How many solutions can the system have?

After substitution you get a quadratic equation, so the discriminant tells the number of intersection points.



A line and a parabola can meet in 2, 1, or 0 points.

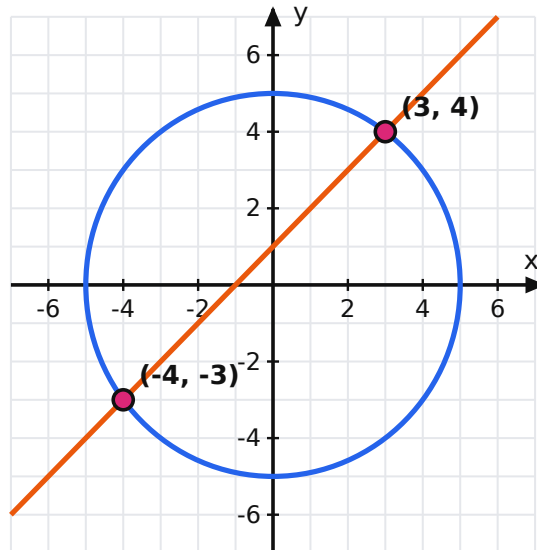
Intersection and discriminant

If $D > 0$, the line cuts the parabola in two points. If $D = 0$, the line is tangent and meets it once. If $D < 0$, the line misses the parabola and the system has no real solution.

For $y = x^2 + 3$ and $y = 2x$: $x^2 - 2x + 3 = 0$ has $D = 4 - 12 = -8$, so there is no real solution. For $y = x^2 + 1$ and $y = 2x$: $x^2 - 2x + 1 = 0$ has $D = 0$, so the line touches at $(1, 2)$.

What about a line and a circle?

The circle $x^2 + y^2 = 25$ is also a quadratic relation. Substitute $y = x + 1$: $x^2 + (x + 1)^2 = 25$ gives $x^2 + x - 12 = 0$ and $x = 3$ or -4 . The points are $(3, 4)$ and $(-4, -3)$.



Circle $x^2 + y^2 = 25$ and line $y = x + 1$ meet at $(3, 4)$ and $(-4, -3)$.

Discriminant D	Intersections	Line is...
$D > 0$	2 points	a secant
$D = 0$	1 point	a tangent
$D < 0$	none	outside

The discriminant decides how many points the line and conic share.

Questions about systems of linear and quadratic equations

How do you solve a linear-quadratic system?

Use substitution. Replace y in the quadratic with the linear expression, solve the new quadratic equation for x , and then find y for each x using the linear equation.

How many solutions can a line and a parabola have?

A line and a parabola can intersect in zero, one, or two points. The number of points matches the discriminant of the quadratic equation you get after substitution.

What does it mean if the discriminant is zero?

The line is tangent to the curve and they share exactly one point, so the system has one real solution.

Can you solve a system by graphing?

Yes. Graph both equations and read the intersection points. Graphing gives estimates, so

algebra is needed to find exact values, especially when the solutions are irrational.

Why do you need to find the y-values?

A solution of a system is an ordered pair (x, y) . Finding only x gives half of each solution.