

Algebra 2 - The Normal Distribution - Lesson

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A normal distribution is a symmetric bell curve described by its mean and standard deviation. About 68% of the data lie within 1 standard deviation of the mean, 95% within 2, and 99.7% within 3. A z-score, $(x - \text{mean}) / \text{standard deviation}$, measures distance from the mean in standard deviations and lets you find percentages.

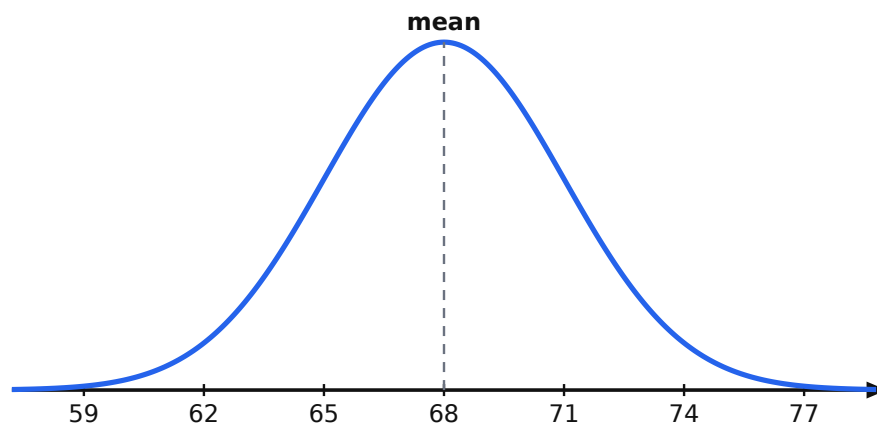
Algebra 2 · Common Core State Standards: **S-ID.A.4**

Key facts

- Normal distribution: symmetric, bell-shaped, described by its mean and standard deviation.
- 68-95-99.7 rule for 1, 2, 3 standard deviations.
- $z = (x - \text{mean}) / \text{sd}$ and $x = \text{mean} + z \text{ times sd}$.
- Area under the curve = proportion of data.
- Compare different distributions with z-scores.

What is a normal distribution?

A **normal distribution** is a symmetric, bell-shaped distribution described by its mean μ and standard deviation σ . The mean is the center, and the standard deviation measures the typical distance from the mean. Many measurements, such as heights, test scores, and the contents of filled bottles, are approximately normal.

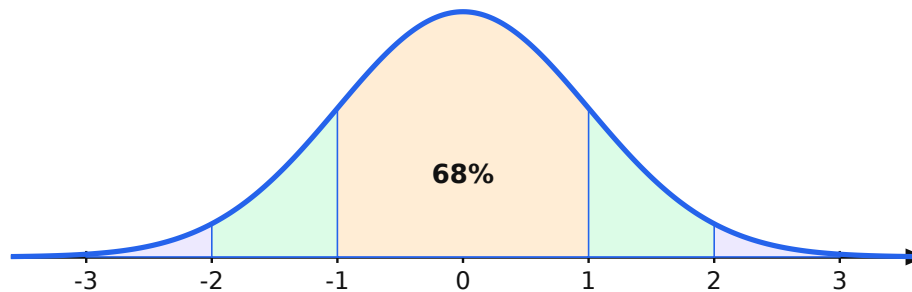


Heights: mean 68 inches, standard deviation 3 inches.

The 68-95-99.7 rule

In a normal distribution, about 68% of the data lie within 1 standard deviation of the mean, about 95% within 2 standard deviations, and about 99.7% within 3 standard deviations.

95% within 2 sd; 99.7% within 3 sd



The 68-95-99.7 rule: z-scores from -3 to 3.

For heights with mean 68 and standard deviation 3, about 68% of people are between $68-3=65$ and $68+3=71$ inches, and about 95% are between 62 and 74 inches. Only about 0.15% are taller than 77 inches, because 77 is 3 standard deviations above the mean and the tails share the remaining 0.3% equally.

What is a z-score?

A **z-score** tells how many standard deviations a value is from the mean: $z = \frac{x-\mu}{\sigma}$. A positive z-score is above the mean and a negative one is below. To compare values from different distributions, compare their z-scores. To go backward, use $x = \mu + z\sigma$.

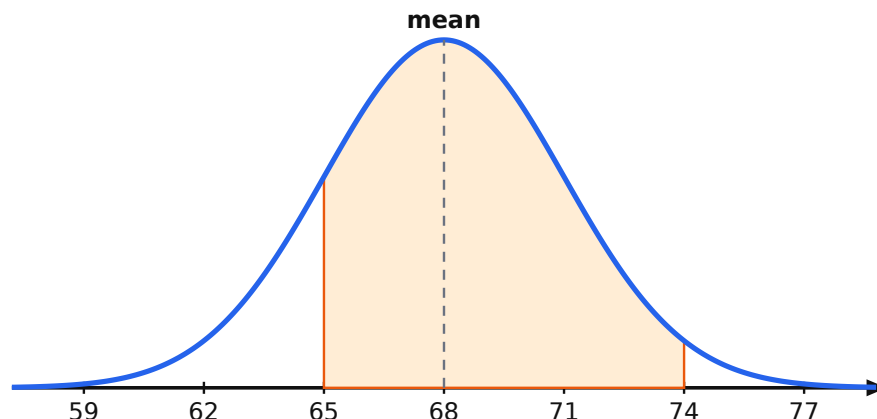
Score x	Mean	SD	z
84	72	6	2
55	70	10	-1.5
620	500	100	1.2

Computing z-scores.

How do you find the percentage of data in an interval?

For values that are not whole standard deviations from the mean, convert to z-scores and use a standard normal table or a calculator. The area under the curve between two values is the proportion of data in that interval. For example, with $z=1$, about 84.1% of data are below the

mean plus one standard deviation. The area between $z = -1$ and $z = 2$ is $0.9772 - 0.1587 = 0.8186$, or about 81.9%.



The shaded region is between $z = -1$ and $z = 2$: about 81.9% of heights.

1. Find the mean and standard deviation.
2. Convert the boundaries to z-scores.
3. Find the area under the standard normal curve between them.
4. Interpret the area as a percent or a count of the population.

Questions about the normal distribution

What does the 68-95-99.7 rule say?

In a normal distribution about 68% of the data are within one standard deviation of the mean, 95% within two, and 99.7% within three.

What is a z-score?

A z-score is the number of standard deviations a value is above or below the mean: $z = (x - \text{mean}) / \text{standard deviation}$.

How do you compare scores from two different tests?

Convert each score to a z-score. The larger z-score is better relative to its own distribution.

What does the area under a normal curve represent?

The area between two values is the proportion of the data that lie between them. The total area under the whole curve is 1.

When can you use a normal model?

Use it when the data are roughly symmetric and bell shaped, with most values near the mean and few far from it. Check that about 68% lie within one standard deviation.