

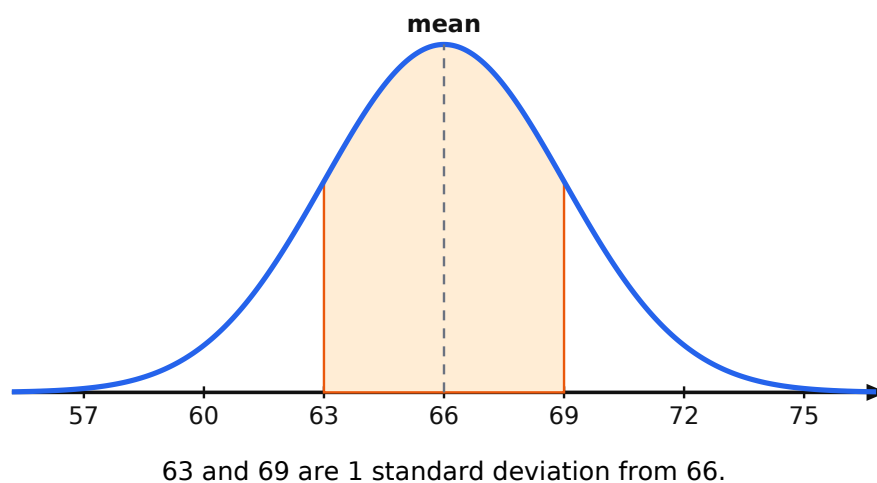
Algebra 2 - The Normal Distribution - Answer Key

Common Core S-ID.A.4 · maths-pdf.fr/en/

Every worksheet problem on the normal distribution is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where confusing standard deviations with percents can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **S-ID.A.4**

Exercise 1 - Within one standard deviation



$66 - 3 = 63$ and $66 + 3 = 69$: within 1 standard deviation.

About 68%

Exercise 2 - Within two standard deviations

$66 \pm 2(3)$ gives 60 and 72.

About 95%

Exercise 3 - Tail percent

$75 = 66 + 3(3)$, so 75 is 3 standard deviations above the mean. 99.7% are within 3, leaving

$100\% - 99.7\% = 0.3\%$ in the two tails.

By symmetry, half of 0.3% is in the upper tail.

About 0.15%

Exercise 4 - Compute a z-score

$$z = \frac{84-72}{6} = 2.$$

$z = 2$

Exercise 5 - Negative z-score

$$z = \frac{55-70}{10} = -1.5.$$

$z = -1.5$: 1.5 standard deviations below the mean

Exercise 6 - From z to a value

$$x = \mu + z\sigma = 500 + 1.2(100) = 620.$$

620

Exercise 7 - Compare two tests

$$\text{Maya: } z = \frac{88-80}{4} = 2. \text{ Jamal: } z = \frac{85-72}{6} \approx 2.17.$$

Jamal did better (z about 2.17 compared with 2)

Exercise 8 - Area below $z = 1$

By a standard normal table or calculator, $P(Z < 1) \approx 0.8413$.

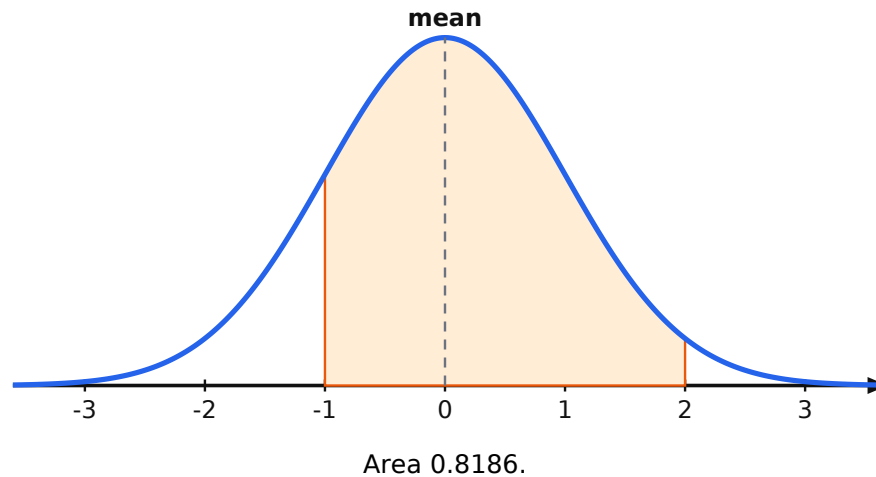
About 84.1%

Exercise 9 - Area above $z = 1.5$

$$P(Z > 1.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668.$$

About 6.68%

Exercise 10 - Area between



$$P(-1 < Z < 2) = 0.9772 - 0.1587 = 0.8186.$$

About 81.9%

Exercise 11 - Percent below a score

$$z = \frac{1250 - 1050}{200} = 1. \quad P(Z < 1) \approx 0.8413.$$

About 84.1%

Exercise 12 - Percent above a score

$$z = \frac{1400 - 1050}{200} = 1.75. \quad P(Z > 1.75) = 1 - 0.9599 = 0.0401.$$

About 4.0%

Exercise 13 - Percent between two scores

$z = -0.75$ and $z = 0.75$. $P(-0.75 < Z < 0.75) = 0.7734 - 0.2266 = 0.5468$.

About 54.7%

Exercise 14 - Percentile

$$x = 1050 + 1.28(200) = 1306.$$

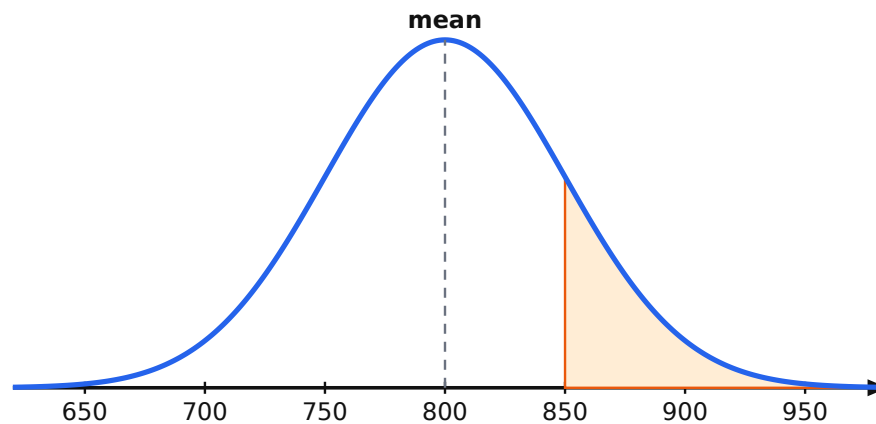
About 1,306

Exercise 15 - Light bulbs

$z = \frac{700 - 800}{50} = -2$. About 2.5% of values are below $z = -2$: $P(Z < -2) \approx 0.0228$.

About 2.3%

Exercise 16 - How many bulbs?



850 is one standard deviation above the mean.

$$z = 1. P(Z > 1) \approx 0.1587.$$

$$2000 \cdot 0.1587 \approx 317.$$

About 317 bulbs

Exercise 17 - Bottle filling

$16.0 \pm 3(0.1)$ gives 15.7 and 16.3, so 99.7% pass.

Rejected: 0.3%. $4000 \cdot 0.003 = 12$.

About 0.3%, which is about 12 bottles

Exercise 18 - Mean and standard deviation

Mean: $\frac{4+6+8+10+12}{5} = 8$.

Squared deviations: 16, 4, 0, 4, 16, total 40, variance $\frac{40}{5} = 8$, so $\sigma = \sqrt{8} \approx 2.83$.

$z = \frac{12-8}{2.83} \approx 1.41$.

Mean 8; standard deviation about 2.83; z about 1.41

Exercise 19 - Is a normal model reasonable?

73 and 83 are within 1 standard deviation of 78. $\frac{17}{25} = 0.68$.

That matches the 68% predicted by the empirical rule.

Yes, 68% is consistent with a normal model

Exercise 20 - Find the error

A z-score counts standard deviations, not percent. A z-score of 2 means the value is 2 standard deviations above the mean, which is $\mu + 2\sigma$.

z = 2 means 2 standard deviations above the mean