

Algebra 2 - Dividing Polynomials and the Remainder Theorem - Lesson

Common Core A-APR.B.2, A-APR.D.6 · maths-pdf.fr/en/

To divide polynomials, divide leading terms, multiply, subtract, and repeat; the result is a quotient plus a remainder over the divisor. When the divisor is $x - k$, synthetic division is a shortcut. The Remainder Theorem says the remainder is $p(k)$, and the Factor Theorem says $x - k$ is a factor exactly when $p(k) = 0$.

Algebra 2 · Common Core State Standards: **A-APR.B.2, A-APR.D.6**

Key facts

- $a(x) / b(x) = q(x) + r(x) / b(x)$ with $\deg r < \deg b$.
- Synthetic division works for divisors $x - k$.
- Remainder Theorem: $p(x) \div (x - k)$ has remainder $p(k)$.
- Factor Theorem: $x - k$ is a factor if and only if $p(k) = 0$.
- Include 0 coefficients for missing powers.

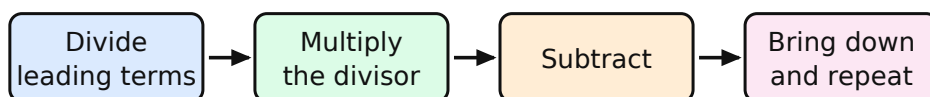
How does polynomial long division work?

Dividing polynomials is like dividing whole numbers: divide, multiply, subtract, and bring down. The result is a **quotient** and a **remainder** whose degree is less than the divisor's degree.

Division of polynomials

For polynomials $a(x)$ and $b(x)$ with $b(x) \neq 0$, there are unique polynomials $q(x)$ and $r(x)$ with $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$, where the degree of $r(x)$ is less than the degree of $b(x)$.

1. Write both polynomials in descending order and include zero terms for missing powers.
2. Divide the leading term of the dividend by the leading term of the divisor.
3. Multiply the divisor by that result and subtract from the dividend.
4. Repeat with the new polynomial until its degree is less than the divisor's.



The long division cycle repeats until the degree of the remainder is smaller than the divisor's.

Example: $(2x^3 + x^2 - 13x + 6) \div (x - 3)$. First $2x^3 \div x = 2x^2$, then $7x$, then 8 , leaving remainder 30 . So the quotient is $2x^2 + 7x + 8$ with remainder 30 .

3	2	1	-13	6
		6	21	24
	2	7	8	30

Synthetic division of $2x^3 + x^2 - 13x + 6$ by $x - 3$: quotient $2, 7, 8$ and remainder 30 .

What is synthetic division?

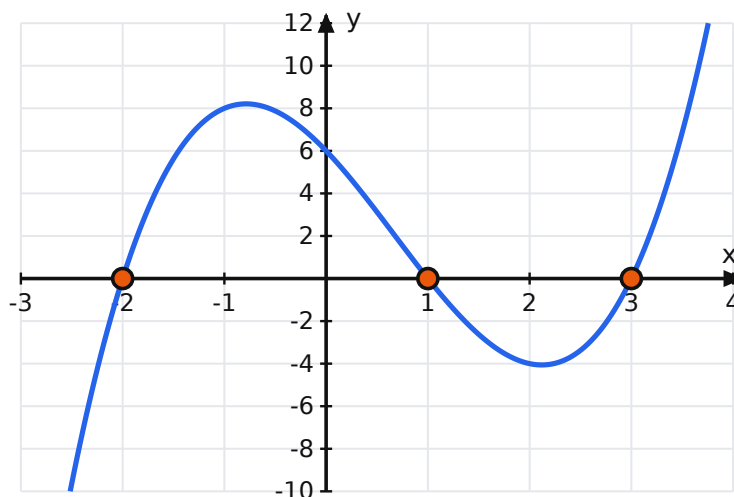
When the divisor has the form $x - k$, you can skip the variables. Write k at the left, bring down the leading coefficient, then multiply by k and add, column by column. The last number is the remainder and the others are the quotient's coefficients. A missing power needs a 0 coefficient.

What is the Remainder Theorem?

Remainder Theorem and Factor Theorem

When a polynomial $p(x)$ is divided by $x - k$, the remainder is $p(k)$. Consequently, $x - k$ is a factor of $p(x)$ if and only if $p(k) = 0$.

In the example, $p(3) = 2(27) + 9 - 39 + 6 = 30$, which matches the remainder. For $p(x) = x^3 - 2x^2 - 5x + 6$, we get $p(1) = 0$, so $x - 1$ is a factor.



$p(x) = x^3 - 2x^2 - 5x + 6$ crosses the x -axis at -2 , 1 , and 3 , so $(x + 2)$, $(x - 1)$, and $(x - 3)$ are factors.

k	p(k)	Factor?
-2	0	$x + 2$: yes
1	0	$x - 1$: yes
2	-4	$x - 2$: no
3	0	$x - 3$: yes

$p(k) = 0$ exactly when $x - k$ is a factor.

Questions about dividing polynomials and the remainder theorem

What is the Remainder Theorem?

If a polynomial $p(x)$ is divided by $x - k$, the remainder equals $p(k)$. You can find the remainder by evaluating the polynomial instead of dividing.

What is the Factor Theorem?

The binomial $x - k$ is a factor of $p(x)$ exactly when $p(k) = 0$. So a zero of a polynomial always matches a linear factor.

When can you use synthetic division?

Use it when dividing by a linear binomial of the form $x - k$. For $x + 3$, use $k = -3$. Include a 0 for each missing power.

What do you do with a remainder?

Write the answer as the quotient plus the remainder over the divisor, for example $x + 4 + 3$ over $(x - 1)$. A remainder of 0 means the divisor is a factor.

Why must you include zero coefficients?

Each column in synthetic division stands for one power of x . Skipping a missing power shifts the coefficients and gives a wrong quotient.