

Algebra 2 - Dividing Polynomials and the Remainder Theorem - Answer Key

Common Core A-APR.B.2, A-APR.D.6 · maths-pdf.fr/en/

Every worksheet problem on polynomial division and the Remainder Theorem is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where a missing zero coefficient can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **A-APR.B.2, A-APR.D.6**

Exercise 1 - Divide, no remainder

1. Divide the leading terms: $x^2 \div x$ gives x . Multiply: $x(x+3) = x^2 + 3x$. Subtract:
 $x^2 + 7x + 12 - (x^2 + 3x) = 4x + 12$.
2. Divide the leading terms: $4x \div x$ gives 4 . Multiply: $4(x+3) = 4x + 12$. Subtract:
 $4x + 12 - (4x + 12) = 0$.

Quotient $x+4$, remainder 0.

Check: $(x+3)(x+4) + 0 = x^2 + 7x + 12$.

$x+4$

Exercise 2 - Divide, remainder

1. Divide the leading terms: $x^2 \div x$ gives x . Multiply: $x(x-2) = x^2 - 2x$. Subtract:
 $x^2 - 5x + 7 - (x^2 - 2x) = -3x + 7$.
2. Divide the leading terms: $-3x \div x$ gives -3 . Multiply: $-3(x-2) = -3x + 6$. Subtract:
 $-3x + 7 - (-3x + 6) = 1$.

Quotient $x-3$, remainder 1.

Check: $(x-2)(x-3) + 1 = x^2 - 5x + 7$.

$x-3 + \frac{1}{x-2}$

Exercise 3 - Leading coefficient 2

1. Divide the leading terms: $10x^2 \div 2x$ gives $5x$. Multiply: $5x(2x+3) = 10x^2 + 15x$. Subtract:
 $10x^2 - x - 24 - (10x^2 + 15x) = -16x - 24$.
2. Divide the leading terms: $-16x \div 2x$ gives -8 . Multiply: $-8(2x+3) = -16x - 24$. Subtract:
 $-16x - 24 - (-16x - 24) = 0$.

Quotient $5x - 8$, remainder 0.

Check: $(2x+3)(5x-8) + 0 = 10x^2 - x - 24$.

$5x - 8$

Exercise 4 - Cubic long division

1. Divide the leading terms: $x^3 \div x$ gives x^2 . Multiply: $x^2(x+1) = x^3 + x^2$. Subtract:
 $x^3 + 2x^2 - 5x - 6 - (x^3 + x^2) = x^2 - 5x - 6$.
2. Divide the leading terms: $x^2 \div x$ gives x . Multiply: $x(x+1) = x^2 + x$. Subtract:
 $x^2 - 5x - 6 - (x^2 + x) = -6x - 6$.
3. Divide the leading terms: $-6x \div x$ gives -6 . Multiply: $-6(x+1) = -6x - 6$. Subtract:
 $-6x - 6 - (-6x - 6) = 0$.

Quotient $x^2 + x - 6$, remainder 0.

Check: $(x+1)(x^2 + x - 6) + 0 = x^3 + 2x^2 - 5x - 6$.

$x^2 + x - 6$

Exercise 5 - Synthetic division

2	2	-3	1	-5
		4	2	6
	2	1	3	1

Synthetic division by $x - 2$: the last number is the remainder.

Bring down 2. Multiply by 2 and add, column by column: 2, 1, 3, 1.

Quotient $2x^2 + x + 3$, remainder 1.

$2x^2 + x + 3 + \frac{1}{x-2}$

Exercise 6 - Missing term

Write the coefficients with a 0 for x^2 : 1, 0, -7, 6.

Bottom row with $k=2$: 1, 2, -3, 0.

Quotient x^2+2x-3 , **remainder** 0

Exercise 7 - Divisor $x + 3$

Coefficients with a zero for x^2 : 1, 3, 0, -1, 4.

Bottom row: 1, 0, 0, -1, 7.

$$x^3-1+\frac{7}{x+3}$$

Exercise 8 - Remainder Theorem, zero

By the Remainder Theorem, the remainder is $p(3)=27-36+3+6$.

$p(3)=0$: remainder 0

Exercise 9 - Remainder Theorem

$x+2 = x-(-2)$, so evaluate $p(-2)=2(-8)+4+10+4$.

Remainder 2

Exercise 10 - Factor check

$$p(4)=64-80+8+8=0.$$

The remainder is 0.

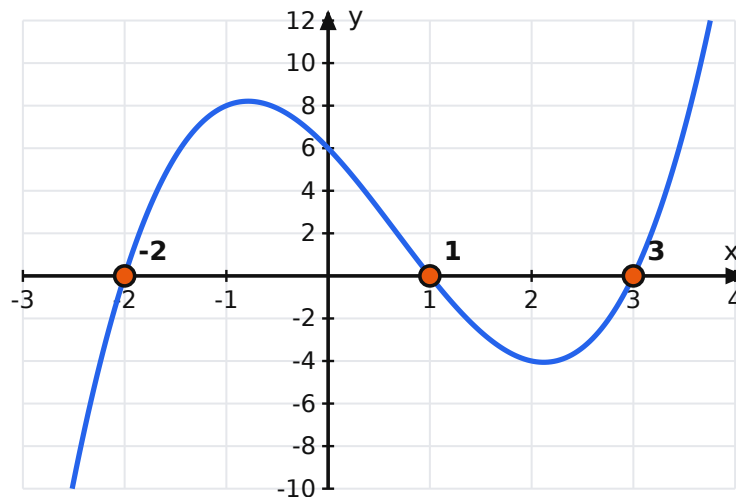
Yes, $x - 4$ is a factor

Exercise 11 - Find k

Since $x+1 = x-(-1)$, set $p(-1) = -1+k+5+3 = k+7$ equal to 0.

$$k = -7$$

Exercise 12 - Read the graph



Zeros -2, 1, 3.

Each zero k gives a factor $x-k$: $x+2$, $x-1$, $x-3$.

Check the leading coefficient: $(x+2)(x-1)(x-3)$ has leading coefficient 1, matching $p(x)$.

$$p(x) = (x+2)(x-1)(x-3)$$

Exercise 13 - Factor completely

1. Divide the leading terms: $x^3 \div x$ gives x^2 . Multiply: $x^2(x-1) = x^3 - x^2$. Subtract:
 $x^3 - 6x^2 + 11x - 6 - (x^3 - x^2) = -5x^2 + 11x - 6$.
2. Divide the leading terms: $-5x^2 \div x$ gives $-5x$. Multiply: $-5x(x-1) = -5x^2 + 5x$. Subtract:
 $-5x^2 + 11x - 6 - (-5x^2 + 5x) = 6x - 6$.
3. Divide the leading terms: $6x \div x$ gives 6. Multiply: $6(x-1) = 6x - 6$. Subtract:
 $6x - 6 - (6x - 6) = 0$.

Quotient $x^2 - 5x + 6$, remainder 0.

$$\text{Check: } (x-1)(x^2 - 5x + 6) + 0 = x^3 - 6x^2 + 11x - 6.$$

$$x^2 - 5x + 6$$

$$\text{Factor the quotient: } x^2 - 5x + 6 = (x-2)(x-3).$$

$$(x-1)(x-2)(x-3)$$

Exercise 14 - Rewrite a rational expression

1. Divide the leading terms: $x^2 \div x$ gives x . Multiply: $x(x+2) = x^2 + 2x$. Subtract: $x^2 + 3x + 5 - (x^2 + 2x) = x + 5$.
2. Divide the leading terms: $x \div x$ gives 1. Multiply: $1(x+2) = x + 2$. Subtract: $x + 5 - (x + 2) = 3$.

Quotient $x+1$, remainder 3.

Check: $(x+2)(x+1)+3 = x^2+3x+5$.

$$x+1+\frac{3}{x+2}$$

Exercise 15 - Rewrite with a cubic

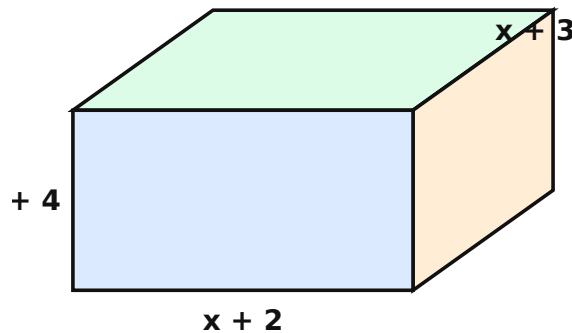
1. Divide the leading terms: $3x^3 \div x$ gives $3x^2$. Multiply: $3x^2(x-1) = 3x^3 - 3x^2$. Subtract: $3x^3 - 2x^2 + 4 - (3x^3 - 3x^2) = x^2 + 4$.
2. Divide the leading terms: $x^2 \div x$ gives x . Multiply: $x(x-1) = x^2 - x$. Subtract: $x^2 + 4 - (x^2 - x) = x + 4$.
3. Divide the leading terms: $x \div x$ gives 1. Multiply: $1(x-1) = x - 1$. Subtract: $x + 4 - (x - 1) = 5$.

Quotient $3x^2+x+1$, remainder 5.

Check: $(x-1)(3x^2+x+1)+5 = 3x^3-2x^2+4$.

$$3x^2+x+1+\frac{5}{x-1}$$

Exercise 16 - Box dimensions



Height found: $x + 4$.

Height = volume divided by length times width: $(x^3+9x^2+26x+24) \div (x^2+5x+6)$.

1. Divide the leading terms: $x^3 \div x^2$ gives x . Multiply: $x(x^2+5x+6) = x^3+5x^2+6x$. Subtract: $x^3+9x^2+26x+24 - (x^3+5x^2+6x) = 4x^2+20x+24$.
2. Divide the leading terms: $4x^2 \div x^2$ gives 4 . Multiply: $4(x^2+5x+6) = 4x^2+20x+24$. Subtract: $4x^2+20x+24 - (4x^2+20x+24) = 0$.

Height $x+4$ inches

Exercise 17 - Rectangle length

1. Divide the leading terms: $x^3 \div x$ gives x^2 . Multiply: $x^2(x+3) = x^3+3x^2$. Subtract: $x^3+3x^2-4x-12 - (x^3+3x^2) = -4x-12$.
2. Divide the leading terms: $-4x \div x$ gives -4 . Multiply: $-4(x+3) = -4x-12$. Subtract: $-4x-12 - (-4x-12) = 0$.

Quotient x^2-4 , remainder 0.

Check: $(x+3)(x^2-4)+0 = x^3+3x^2-4x-12$.

x^2-4

Exercise 18 - Evaluate with synthetic division

4	1	-6	11	2
		4	-8	12
	1	-2	3	14

Last entry: remainder = $P(4) = 14$.

Bottom row with $k=4$: 1, -2, 3, 14. By the Remainder Theorem $P(4)=14$.

1,400 individuals at $t = 4$

Exercise 19 - Difference of cubes

$p(5) = 125 - 125 = 0$, so $x-5$ is a factor.

Divide: $x^3-125 = (x-5)(x^2+5x+25)$.

The quadratic has discriminant $25-100 = -75 < 0$, so it does not factor over the real numbers.

$$(x-5)(x^2+5x+25)$$

Exercise 20 - No division needed

$$p(1)=1-1=0. \quad p(-1)=(-1)^{10}-1=1-1=0.$$

Both remainders are 0