

Algebra 2 - Polynomial Operations and Identities

- Lesson

Common Core A-APR.A.1, A-APR.C.4 · maths-pdf.fr/en/

A polynomial is a sum of terms with whole-number exponents, and sums, differences, and products of polynomials are again polynomials. Add by combining like terms and multiply by distributing every term. Identities such as the difference of squares, $(a + b)(a - b) = a^2 - b^2$, hold for all values and speed up algebra and mental math.

Algebra 2 · Common Core State Standards: **A-APR.A.1, A-APR.C.4**

Key facts

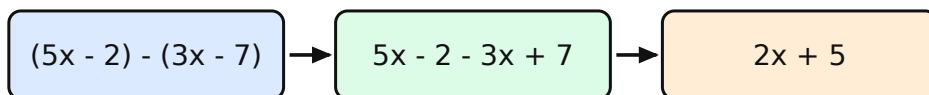
- Polynomials are closed under addition, subtraction, and multiplication.
- Combine like terms; subtract by changing every sign.
- Degree of a product = sum of the degrees.
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$ and $(a + b)(a - b) = a^2 - b^2$.
- $(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$ makes Pythagorean triples.

Why are polynomials closed under addition and multiplication?

A **polynomial** is a sum of terms like ax^n with whole-number exponents. Adding, subtracting, or multiplying two polynomials always gives another polynomial. This property is called **closure**, the same way integers are closed under addition and multiplication but not under division.

Operations on polynomials

To add or subtract, combine like terms (same variable, same power). To multiply, distribute every term of one factor over every term of the other, then combine like terms. The degree of a product is the sum of the degrees.



Distribute the minus sign, then combine like terms.

For example, $(2x^2+3x-1)+(x^2-5x+4)=3x^2-2x+3$. When subtracting, change the sign of every term in the second polynomial first.

	x	3
x	x^2	$3x$
2	$2x$	6

$(x + 3)(x + 2)$: the cells x^2 , $2x$, $3x$, and 6 add to $x^2 + 5x + 6$.

Which products are worth memorizing?

Some patterns appear so often that they are treated as identities, equations true for all values of the variables.

Identity	Name
$(a+b)^2 = a^2 + 2ab + b^2$	Square of a sum
$(a-b)^2 = a^2 - 2ab + b^2$	Square of a difference
$(a+b)(a-b) = a^2 - b^2$	Difference of squares
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	Cube of a sum

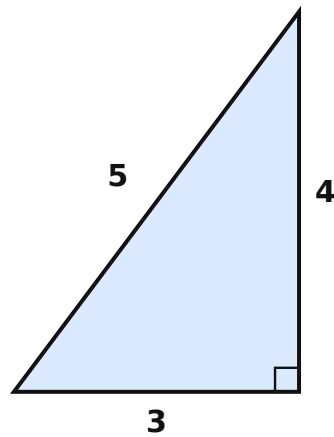
	x	3
x	x^2	$x \cdot 3$
3	$x \cdot 3$	3^2

The square of side $a + b$ has total area $a^2 + 2ab + b^2$.

A common mistake is $(a+b)^2 = a^2 + b^2$. The picture shows the two missing rectangles, which together are $2ab$.

How can identities be used for mental math and number patterns?

Write numbers as sums or differences: $47^2 = (50-3)^2 = 2500 - 300 + 9 = 2209$ and $53 \cdot 47 = (50+3)(50-3) = 2500 - 9 = 2491$. Identities also generate Pythagorean triples. For positive integers $x > y$: $(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$. With $x=2$ and $y=1$, the sides are 3, 4, and 5.



$x = 2, y = 1$ gives the triple 3, 4, 5. With $x = 3, y = 2$ the triple is 5, 12, 13.

1. Choose two positive integers $x > y$.
2. Compute the legs $x^2 - y^2$ and $2xy$.
3. Compute the hypotenuse $x^2 + y^2$.

Questions about polynomial operations and identities

What does it mean that polynomials are closed under addition?

Adding two polynomials always gives another polynomial. The result may have a smaller degree, but it never has a negative or fractional exponent.

How do you subtract polynomials?

Change the sign of every term in the second polynomial, then combine like terms. For example, $(5x - 2) - (3x - 7) = 5x - 2 - 3x + 7 = 2x + 5$.

Is $(a + b)$ squared equal to a squared plus b squared?

No. The correct expansion is a squared plus $2ab$ plus b squared. The middle term $2ab$ is the area of two rectangles in the square model.

What is a polynomial identity?

An identity is an equation that is true for every value of the variables, such as the difference of squares $(a + b)(a - b) = a$ squared minus b squared.

How do you generate a Pythagorean triple?

Pick integers x greater than y . Then $x^2 - y^2$, $2xy$, and $x^2 + y^2$ form a right triangle. For $x = 3$, $y = 2$ the triple is 5, 12, 13.