

Algebra 2 - Zeros and Graphs of Polynomial Functions - Lesson

Common Core A-APR.B.3, F-IF.C.7c · maths-pdf.fr/en/

A zero of a polynomial is an input where the output is 0, and it appears as an x-intercept of the graph. Factor, set each factor equal to zero, and solve. Odd multiplicity crosses the axis and even multiplicity touches it, while the degree and leading coefficient set the end behavior.

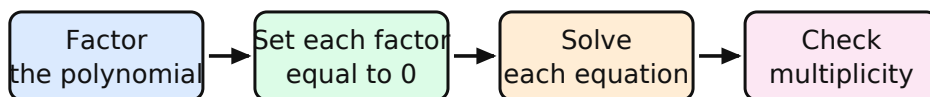
Algebra 2 · Common Core State Standards: **A-APR.B.3, F-IF.C.7c**

Key facts

- Zeros of $p(x)$ are x-intercepts; $x - r$ is a factor when $p(r) = 0$.
- Odd multiplicity: the graph crosses; even multiplicity: it touches and turns.
- End behavior follows the leading term $a \cdot x^n$.
- Rational Root Theorem: candidates are $\pm$ (factors of constant) / (factors of leading coefficient).
- A degree n polynomial has n complex zeros counting multiplicity.

What is a zero of a polynomial?

A **zero** of $p(x)$ is a number r with $p(r) = 0$. Zeros are the x-intercepts of the graph. By the Factor Theorem, $x - r$ is then a factor. The most reliable way to find zeros is to factor, set each factor equal to zero, and solve.



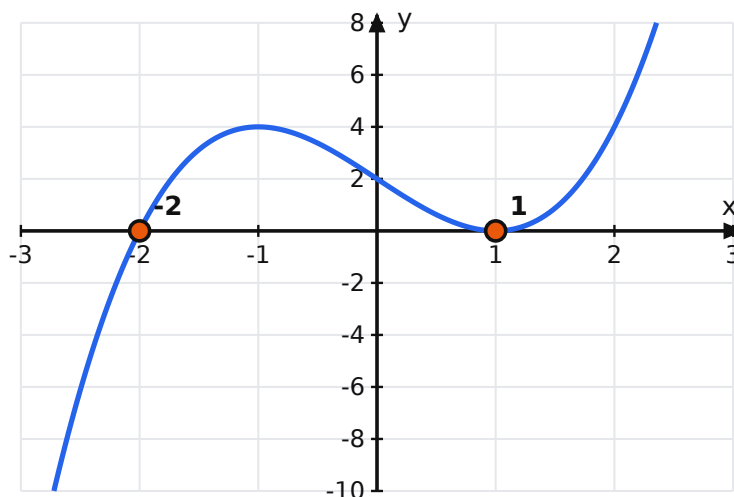
Finding zeros by factoring.

Use a greatest common factor, grouping, or substitution for quadratic type expressions. For example, $x^3 - 9x = x(x-3)(x+3)$ has zeros 0, 3, and -3 . And $x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4)$ has zeros ± 1 and ± 2 .

What does multiplicity tell you about the graph?

Multiplicity

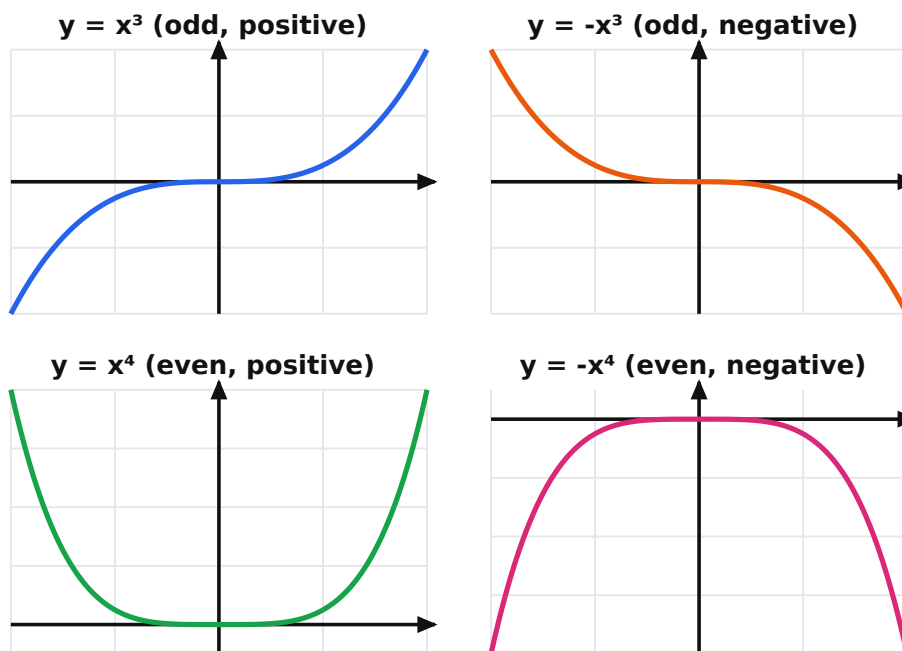
If $(x-r)^k$ is a factor and $(x-r)^{k+1}$ is not, r is a zero of **multiplicity k** . If k is odd, the graph crosses the x-axis at r . If k is even, the graph touches the axis and turns around.



$y = (x + 2)(x - 1)^2$: crosses at -2 (multiplicity 1) and touches at 1 (multiplicity 2).

How do you predict the end behavior?

For large $|x|$, a polynomial behaves like its leading term $a_n x^n$.



End behavior depends on the degree (even or odd) and the sign of the leading coefficient.

Degree	Leading coefficient	Left end	Right end
Odd	Positive	Down	Up

Degree	Leading coefficient	Left end	Right end
Odd	Negative	Up	Down
Even	Positive	Up	Up
Even	Negative	Down	Down

How do you find zeros that do not factor easily?

The **Rational Root Theorem** says that a rational zero of a polynomial with integer coefficients has the form $\frac{p}{q}$, where p divides the constant term and q divides the leading coefficient. For $2x^3 - 3x^2 - 11x + 6$, the candidates are $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$. Test each candidate with the Remainder Theorem; $x = 3$ works, then divide to get the quadratic factor.

Candidate r	$p(r)$	Zero?
1	-6	no
-2	0	yes
3	0	yes
$1/2$	0	yes

Testing candidates: zeros are -2, 3, and $1/2$.

Questions about zeros and graphs of polynomial functions

What is a zero of a polynomial function?

A zero is an input that makes the function equal 0. On the graph it is an x-intercept, and it matches a linear factor of the polynomial.

What does multiplicity mean?

Multiplicity is the number of times a factor repeats. An odd multiplicity means the graph crosses the x-axis there, and an even multiplicity means it touches the axis and turns.

How do you find the end behavior of a polynomial?

Look at the degree and the leading coefficient. Even degree with positive leading coefficient rises on both ends. Odd degree with positive leading coefficient falls on the left and rises on the right.

What is the Rational Root Theorem?

It lists the possible rational zeros of a polynomial with integer coefficients: the factors of the

constant term divided by the factors of the leading coefficient.

How many zeros can a polynomial have?

A polynomial of degree n has at most n real zeros and exactly n complex zeros counted with multiplicity. Complex zeros of real polynomials come in conjugate pairs.