

Algebra 2 - Zeros and Graphs of Polynomial Functions - Answer Key

Common Core A-APR.B.3, F-IF.C.7c · maths-pdf.fr/en/

Every worksheet problem on zeros and graphs of polynomial functions is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where a flipped sign when turning a zero into a factor can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **A-APR.B.3, F-IF.C.7c**

Exercise 1 - Zeros from factored form

Set each factor equal to 0: $x-3=0$, $x+5=0$, $2x-1=0$.

$$x = 3, -5, \frac{1}{2}$$

Exercise 2 - Factor out an x

$$x^3 - 9x = x(x^2 - 9) = x(x-3)(x+3).$$

$$x = 0, 3, -3$$

Exercise 3 - Trinomial with a common factor

$$x(x^2 + 2x - 8) = x(x+4)(x-2).$$

$$x = 0, -4, 2$$

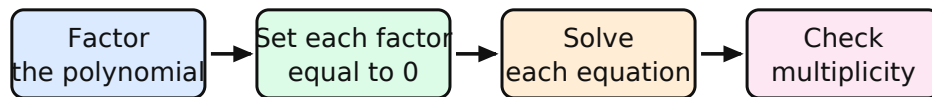
Exercise 4 - Factor by grouping

$$\text{Group: } x^2(x+3) - 4(x+3) = (x+3)(x^2 - 4).$$

$$(x+3)(x-2)(x+2).$$

$$x = -3, 2, -2$$

Exercise 5 - Quadratic type



Finding zeros by factoring.

Let $u = x^2$: $u^2 - 5u + 4 = (u-1)(u-4)$.

So $x^2 = 1$ or $x^2 = 4$.

$x = \pm 1, \pm 2$

Exercise 6 - Multiplicity

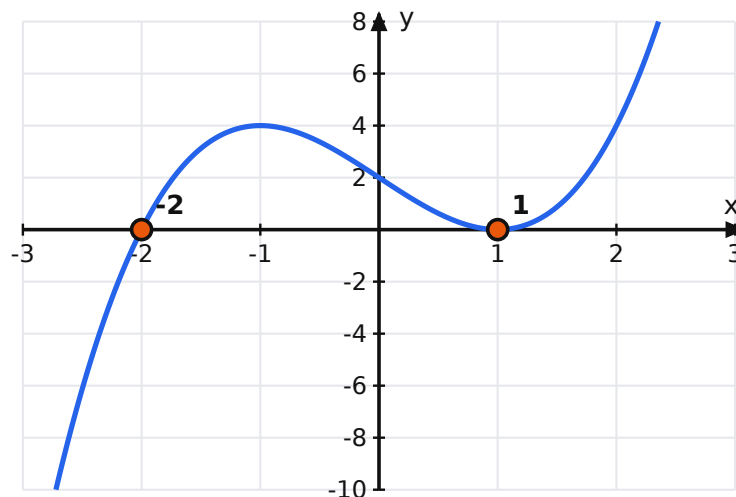
$x = -1$ has multiplicity 2 (even): the graph touches the axis.

$x = 2$ has multiplicity 3 (odd): the graph crosses the axis.

The degree is $2+3=5$.

-1: multiplicity 2, touches; 2: multiplicity 3, crosses

Exercise 7 - Read a graph



Crossing at -2, touching at 1.

The graph crosses at -2 (multiplicity 1) and touches at 1 (multiplicity 2). The total degree is 3.

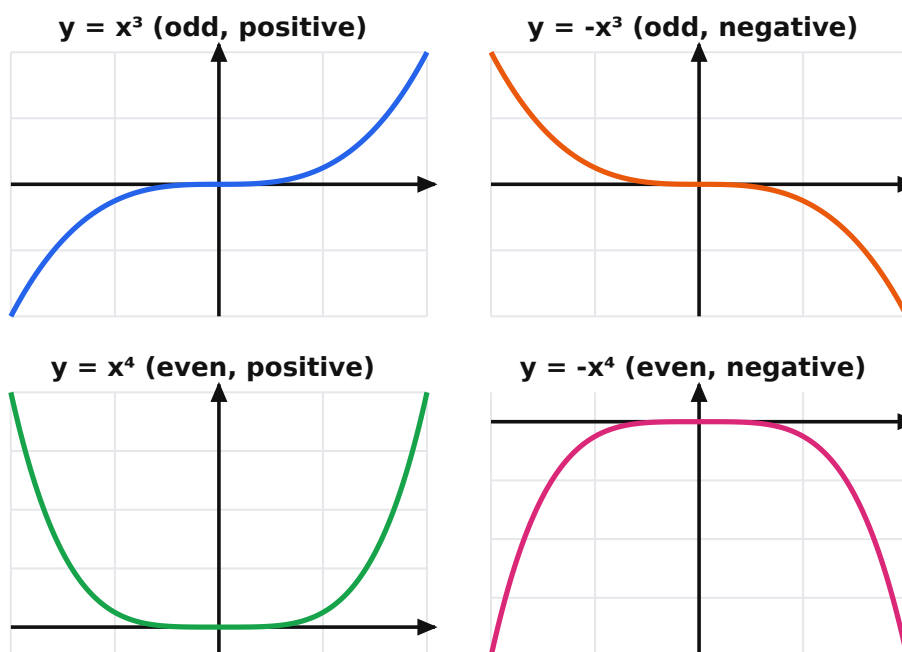
$$f(x) = (x+2)(x-1)^2$$

Exercise 8 - End behavior, odd

The leading term is $-2x^5$: odd degree, negative coefficient.

As x decreases without bound, $f(x)$ increases without bound; as x increases without bound, $f(x)$ decreases without bound

Exercise 9 - End behavior, even



End behavior depends on the degree (even or odd) and the sign of the leading coefficient.

The leading term $3x^4$ has even degree and a positive coefficient.

Both ends rise: $f(x)$ increases without bound as x goes to either end

Exercise 10 - Rational root candidates

Factors of the constant term 6: $\pm 1, \pm 2, \pm 3, \pm 6$. Factors of the leading coefficient 2: $\pm 1, \pm 2$.

Divide each by 1 and 2.

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$$

Exercise 11 - Find all zeros, monic

Candidates: $\pm 1, \pm 2, \pm 3, \pm 6$. $p(1) = 1 - 2 - 5 + 6 = 0$, so $x - 1$ is a factor.

Divide: $x^3 - 2x^2 - 5x + 6 = (x - 1)(x^2 - x - 6) = (x - 1)(x - 3)(x + 2)$.

$$x = 1, 3, -2$$

Exercise 12 - Find all zeros, non-monic

$p(1) = -6$, no. $p(3) = 54 - 27 - 33 + 6 = 0$, so $x - 3$ is a factor.

Synthetic division gives $2x^2 + 3x - 2 = (2x - 1)(x + 2)$.

$$x = 3, \frac{1}{2}, -2$$

Exercise 13 - Build from zeros

$$f(x) = (x - 2)(x + 1)(x - 4).$$

$(x - 2)(x + 1) = x^2 - x - 2$. Multiply by $x - 4$.

$$x^3 - 5x^2 + 2x + 8$$

Exercise 14 - Double zero and a point

$f(x) = a(x - 1)^2(x + 3)$. At $x = 0$: $a(1)(3) = 6$, so $a = 2$.

Expand $2(x^2 - 2x + 1)(x + 3)$.

$$2x^3 + 2x^2 - 10x + 6$$

Exercise 15 - Sign chart

Interval	$x < -1$	$-1 < x < 2$	$2 < x < 4$	$x > 4$
Sign of f	negative	positive	negative	positive

Signs alternate at each zero of multiplicity 1.

Test points: $f(-2) = (-1)(-4)(-6) = -24$, $f(0) = (1)(-2)(-4) = 8$, $f(3) = (4)(1)(-1) = -4$,

$$f(5) = (6)(3)(1) = 18.$$

negative, positive, negative, positive

Exercise 16 - Complex zeros

$$x^3 - 1 = (x - 1)(x^2 + x + 1). \text{ The first factor gives } x = 1.$$

$$\text{Quadratic formula: } x = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}.$$

$$x = 1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$$

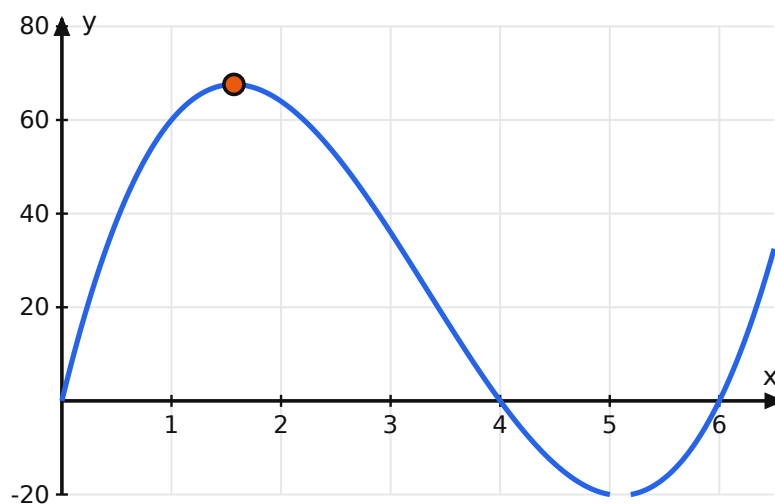
Exercise 17 - Conjugate zeros

Complex zeros of a real polynomial come in conjugate pairs, so $-3i$ is also a zero.

Count: $2 + 1 + 1 + 1 = 5$, so there are no others.

$$-3i$$

Exercise 18 - Open box



Maximum volume about 67.6 cubic inches at x about 1.57.

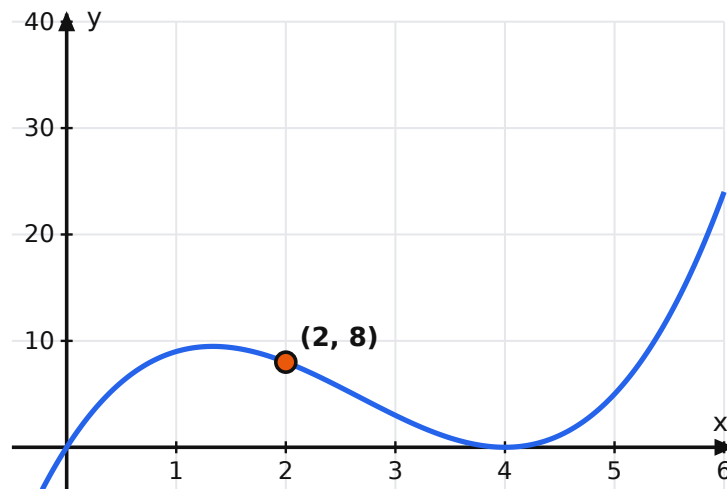
$$V(x) = x(12 - 2x)(8 - 2x), \text{ so the zeros are } x = 0, 6, 4.$$

A cut cannot exceed half of the shorter side (4 inches), so only $0 < x < 4$ makes a box.

The maximum is at $x \approx 1.57$ inches, where $V \approx 67.6$ cubic inches.

Zeros 0, 4, 6; domain $0 < x < 4$; best cut about 1.57 in

Exercise 19 - Roller coaster



$h(2) = 8$ tens of feet.

Zeros: $x=0$ (multiplicity 1) and $x=4$ (multiplicity 2).

At $x=4$ the graph touches the ground and turns back up. $h(2) = 2(-2)^2 = 8$.

Ground at $x = 0$ and $x = 4$ (touches at 4); $h(2) = 8$ tens of feet = 80 ft

Exercise 20 - Find the error

The factor $x-2$ is squared, so $x=2$ has even multiplicity 2. The graph touches the axis and turns around, it does not cross.

Maya is wrong; the graph touches at $x = 2$ and crosses at $x = -1$

Exercise 21 - Odd degree always crosses

The ends of an odd-degree graph go in opposite directions: one end goes to positive infinity and the other to negative infinity.

A continuous graph that goes from below the x-axis to above it must cross the axis at least once.

At least one x-intercept, hence one real zero