

Algebra 2 - Conditional Probability and Independence - Lesson

Common Core S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7, S-CP.B.8, S-CP.B.9 · maths-pdf.fr/en/

Conditional probability, $P(A \text{ given } B) = P(A \text{ and } B) / P(B)$, is the probability of an event when another event is known to occur. Events are independent when $P(A \text{ and } B) = P(A) \cdot P(B)$. Use the addition rule, the multiplication rule, two-way tables, tree diagrams, and permutations and combinations to find probabilities.

Algebra 2 · Common Core State Standards: S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7, S-CP.B.8, S-CP.B.9

Key facts

- $P(\text{not } A) = 1 - P(A)$; $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
- $P(A \text{ given } B) = P(A \text{ and } B) / P(B)$.
- Independent: $P(A \text{ and } B) = P(A) \cdot P(B)$.
- Multiplication rule: $P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$.
- Permutations $P(n, r) = n! / (n - r)!$; combinations $C(n, r) = n! / (r!(n - r)!)$.

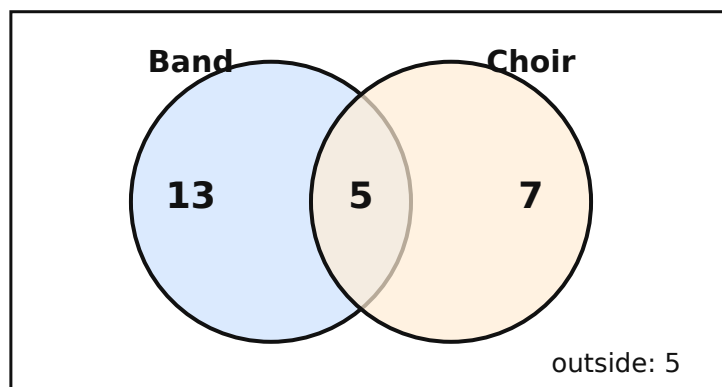
How do you describe events and find their probabilities?

The **sample space** is the set of all possible outcomes, and an **event** is a subset of the sample space. For equally likely outcomes, $P(A) = \frac{\text{number of outcomes in } A}{\text{total outcomes}}$. For two dice, there are 36 outcomes, and 6 of them have sum 7, so $P(\text{sum is } 7) = \frac{6}{36} = \frac{1}{6}$.

	First die					
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12
	1	2	3	4	5	6

All 36 outcomes for two dice; the sums of 7 are highlighted.

The **complement** of A is "not A ": $P(\text{not } A) = 1 - P(A)$. The event " A or B " is the **union** and " A and B " is the **intersection**. The addition rule is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. For example, among 52 cards, $P(\text{heart or king}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$.



30 students: 18 in band (13 + 5), 12 in choir (5 + 7), 5 in both.

What is conditional probability?

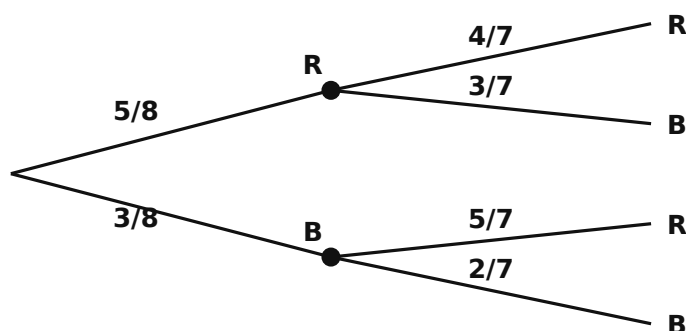
Conditional probability and independence

The **conditional probability** of A given B is $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$, the probability of A when you restrict attention to B . Events are **independent** if knowing one does not change the other: $P(A \text{ and } B) = P(A) \cdot P(B)$, equivalently $P(A|B) = P(A)$.

	Grade 10	Grade 11	Total
Plays a sport	30	20	50
No sport	50	100	150
Total	80	120	200

Of 200 students, 80 play a sport. $P(\text{plays a sport} \mid \text{Grade 11}) = 20/120$.

In the table, $P(\text{sport} \mid \text{Grade 11}) = \frac{20}{120} = \frac{1}{6}$, while $P(\text{sport}) = \frac{50}{200} = \frac{1}{4}$. They are different, so the events are not independent. The multiplication rule is $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.



Two draws from a bag of 5 red (R) and 3 blue (B) marbles, without replacement.

Drawing two marbles without replacement: $P(\text{both red}) = \frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$. With replacement the draws are independent and the probability is $\frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$.

How do you count arrangements and selections?

When order matters, count **permutations**: $P(n, r) = \frac{n!}{(n-r)!}$. Choosing 1st, 2nd, and 3rd place from 5 runners gives $P(5, 3) = 5 \cdot 4 \cdot 3 = 60$. When order does not matter, count **combinations**: $C(n, r) = \frac{n!}{r!(n-r)!}$. A team of 3 chosen from 8 people has $C(8, 3) = 56$ possibilities. Use counting to find probabilities: the probability of one specific team is $\frac{1}{56}$.

Questions about conditional probability and independence

What is conditional probability?

Conditional probability is the probability of an event given that another event has occurred. $P(A \text{ given } B)$ equals $P(A \text{ and } B)$ divided by $P(B)$.

How do you tell if two events are independent?

Check whether $P(A \text{ and } B)$ equals $P(A)$ times $P(B)$. Equivalent: $P(A \text{ given } B)$ equals $P(A)$.

What is the addition rule?

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Subtract the overlap so it is not counted twice.

What is the difference between permutations and combinations?

A permutation counts arrangements where order matters, such as first, second, third place. A combination counts selections where order does not matter, such as a committee.

How do you use a two-way table to find probabilities?

Divide the count in the cell by the total for a joint probability. For a conditional probability, divide by the total of the given row or column.