

Algebra 2 - Conditional Probability and Independence - Answer Key

Common Core S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7, S-CP.B.8, S-CP.B.9 · maths-pdf.fr/en/

Every worksheet problem on conditional probability and independence is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where treating dependent events as independent can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7, S-CP.B.8, S-CP.B.9**

Exercise 1 - Sample space

Outcomes: HH, HT, TH, TT. Exactly one head: HT and TH.

$$\frac{2}{4} = \frac{1}{2}$$

Exercise 2 - Two dice

		First die					
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12
		1	2	3	4	5	6

All 36 outcomes for two dice; the sums of 7 are highlighted.

Six outcomes have sum 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1).

$$\frac{6}{36} = \frac{1}{6}.$$

$$\frac{1}{6}$$

Exercise 3 - Complement

$$1 - \frac{1}{6} = \frac{5}{6}.$$

$$\frac{5}{6}$$

Exercise 4 - Union of events

$$P(\text{heart}) = \frac{13}{52}, P(\text{king}) = \frac{4}{52}, P(\text{king of hearts}) = \frac{1}{52}.$$

$$\frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13}.$$

$$\frac{4}{13}$$

Exercise 5 - Mutually exclusive

Multiples of 5: 5, 10, 15, 20 (4). Multiples of 7: 7, 14 (2). No overlap.

$$\frac{4+2}{20} = \frac{3}{10}.$$

$$\frac{3}{10}$$

Exercise 6 - Independent events

The events are independent: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$

$$\frac{1}{4}$$

Exercise 7 - Test independence

$P(A) \cdot P(B) = 0.4 \cdot 0.5 = 0.2$, which equals $P(A \text{ and } B).$

Yes, they are independent

Exercise 8 - Read a two-way table

	Grade 10	Grade 11	Total
Plays a sport	30	20	50
No sport	50	100	150
Total	80	120	200

Row and column totals.

Plays a sport: $\frac{50}{200}$. Grade 11: $\frac{120}{200}$.

$\frac{1}{4}$ and $\frac{3}{5}$

Exercise 9 - Conditional probability from a table

Restrict to Grade 11: 120 students, of whom 20 play a sport.

$$\frac{20}{120} = \frac{1}{6}.$$

$$\frac{1}{6}$$

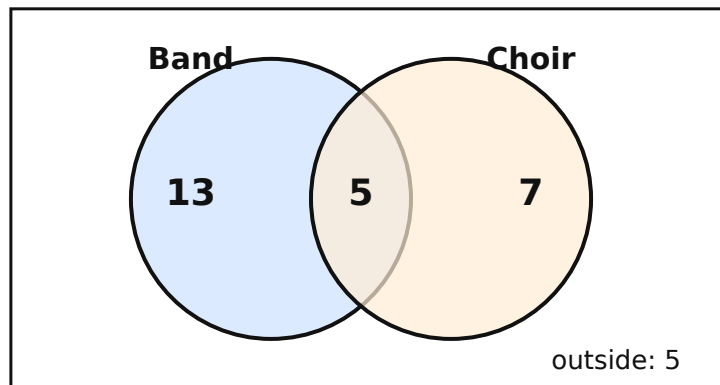
Exercise 10 - Independence in a table

$$P(\text{sport}) = \frac{50}{200} = \frac{1}{4} \text{ but } P(\text{sport}|\text{Grade 11}) = \frac{1}{6}.$$

The probabilities differ.

Not independent

Exercise 11 - Venn diagram

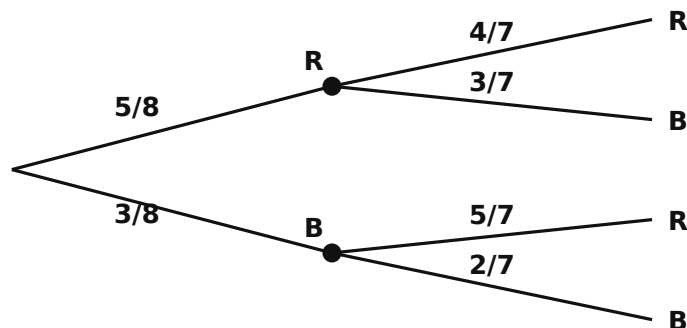


$13 + 5 + 7 = 25$ in band or choir.

$$P(\text{band or choir}) = \frac{18}{30} + \frac{12}{30} - \frac{5}{30} = \frac{25}{30}.$$

$$\frac{5}{6}$$

Exercise 12 - Without replacement



Two draws from a bag of 5 red (R) and 3 blue (B) marbles, without replacement.

$$P(\text{first red}) = \frac{5}{8}. \text{ Then 4 red remain among 7: } P(\text{second red} | \text{first red}) = \frac{4}{7}.$$

$$\frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56} = \frac{5}{14}.$$

$$\frac{5}{14}$$

Exercise 13 - With replacement

$$\text{The draws are independent: } \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}.$$

$$\frac{25}{64}$$

Exercise 14 - One of each color

Red then blue: $\frac{5}{8} \cdot \frac{3}{7} = \frac{15}{56}$. Blue then red: $\frac{3}{8} \cdot \frac{5}{7} = \frac{15}{56}$.

Add: $\frac{30}{56} = \frac{15}{28}$.

$$\frac{15}{28}$$

Exercise 15 - Permutations

Order matters: $P(5,3) = 5 \cdot 4 \cdot 3 = 60$.

60

Exercise 16 - Combinations

Order does not matter: $C(8,3) = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$.

56

Exercise 17 - Committee probability

Favorable: $C(4,2) \cdot C(6,1) = 6 \cdot 6 = 36$. Total: $C(10,3) = 120$.

$$\frac{36}{120} = \frac{3}{10}$$

$$\frac{3}{10}$$

Exercise 18 - Medical test

Positives: $95 + 495 = 590$. Of them, 95 have the disease.

$$\frac{95}{590} \approx 0.161.$$

About 16%, even though the test is accurate

Exercise 19 - At least one

Complement: no 6 in three rolls: $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$.

$$1 - \frac{125}{216} = \frac{91}{216}.$$

$$\frac{91}{216}$$

Exercise 20 - Find the error

Mutually exclusive means $P(A \text{ and } B) = 0$. Independent would need $0.3 \cdot 0.4 = 0.12$. Since $0 \neq 0.12$, the events are not independent.

Mutually exclusive events with positive probabilities are not independent

Exercise 21 - Addition rule

$$0.7 = 0.5 + 0.4 - P(A \text{ and } B), \text{ so } P(A \text{ and } B) = 0.2.$$

0.2