

Algebra 2 - Solving Quadratic Equations - Lesson

Common Core A-REI.B.4a, A-REI.B.4b · maths-pdf.fr/en/

A quadratic equation can be solved by square roots, factoring, completing the square, or the quadratic formula $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$. The discriminant $b^2 - 4ac$ shows whether there are two real, one real, or two complex solutions. Choose the quickest method, and check each solution in the original equation.

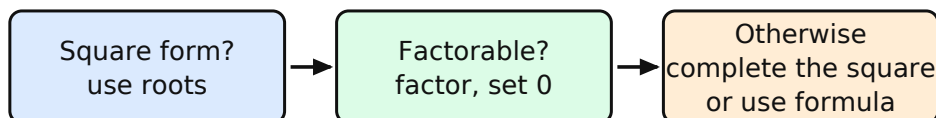
Algebra 2 · Common Core State Standards: **A-REI.B.4a, A-REI.B.4b**

Key facts

- Square roots: isolate the square, then take $\pm$ the root.
- Factoring: use the zero product property.
- Quadratic formula: $x = (-b \pm \sqrt{b^2 - 4ac}) / (2a)$.
- Discriminant > 0 : two real; $= 0$: one real; < 0 : two complex.
- Check solutions, and reject those that do not fit the context.

Which method should you use to solve a quadratic equation?

A quadratic equation $ax^2+bx+c=0$ can be solved four ways. Each method works for some equations, and the quadratic formula works for all of them.



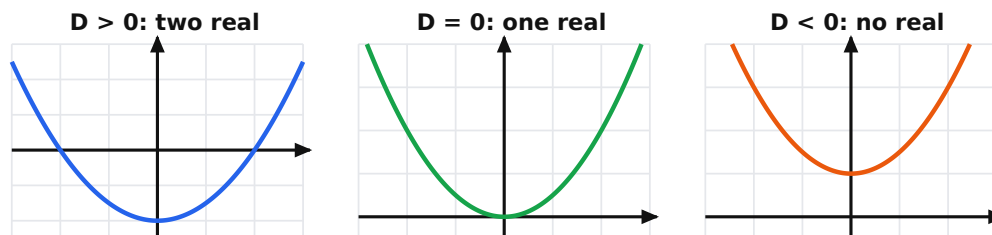
Choose a method.

- **Square roots:** if there is no x -term, isolate the square. $(x-3)^2=16$ gives $x-3=\pm 4$.
- **Factoring:** write as a product equal to zero and use the zero product property.
 $x^2-5x+6=(x-2)(x-3)=0$.
- **Completing the square:** works for every quadratic and leads to the formula.
- **Quadratic formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The discriminant

The expression $D=b^2-4ac$ under the radical is the **discriminant**. If $D>0$, there are two real solutions. If $D=0$, there is one real solution. If $D<0$, there are two complex (non-real)

solutions.

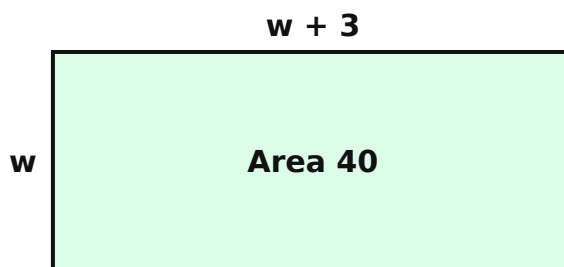


The sign of the discriminant tells how many times the parabola meets the x-axis.

How do you solve by completing the square?

1. Move the constant to the right side.
2. If $a \neq 1$, divide every term by a .
3. Add $\left(\frac{b}{2}\right)^2$ to both sides.
4. Factor the left side as a square and take square roots.

Example: $x^2 - 6x - 1 = 0$. Then $x^2 - 6x = 1$, so $x^2 - 6x + 9 = 10$, giving $(x-3)^2 = 10$ and $x = 3 \pm \sqrt{10}$.



Length is 3 more than the width: area 40 gives $w(w + 3) = 40$.

How do you use the quadratic formula?

Write the equation in standard form and identify a , b , and c . For $2x^2 + 4x - 3 = 0$, $D = 16 + 24 = 40$ and $x = \frac{-4 \pm \sqrt{40}}{4} = \frac{-2 \pm \sqrt{10}}{2}$. Always check by substituting, and simplify the radical and the fraction. For application problems, reject solutions that do not make sense, such as negative lengths.

Equation	D	Solutions
$x^2 - 4x + 4 = 0$	0	2 (repeated)
$x^2 - x - 6 = 0$	25	3 and -2
$x^2 + 2x + 5 = 0$	-16	$-1 \pm 2i$

The discriminant predicts the type of solutions.

Questions about solving quadratic equations

What is the quadratic formula?

For $ax^2 + bx + c = 0$ the solutions are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. It works for every quadratic equation, including those with irrational or complex solutions.

What does the discriminant tell you?

The discriminant $b^2 - 4ac$ tells the number and type of solutions: positive means two real, zero means one real (repeated), and negative means two complex solutions.

When should you complete the square?

Complete the square when the equation does not factor easily and the x-coefficient is even, or when you want vertex form. The quadratic formula comes from completing the square on the general equation.

Why do you write $\pm$ when taking a square root?

Both a positive and a negative number give the same square. For example, both 4 and -4 square to 16, so $x^2 = 16$ has two solutions.

How do you check solutions of a quadratic?

Substitute each solution into the original equation. For word problems, also check that the solution makes sense, for example a length cannot be negative.