

Algebra 2 - Forms of Quadratic Functions - Lesson

Common Core A-SSE.B.3a, A-SSE.B.3b, F-IF.C.8a · maths-pdf.fr/en/

A quadratic function can be written in standard form, factored form, or vertex form, and each form reveals a different feature. Standard form shows the y-intercept, factored form shows the zeros, and vertex form $a(x - h)^2 + k$ shows the vertex. Completing the square converts standard form to vertex form.

Algebra 2 · Common Core State Standards: **A-SSE.B.3a, A-SSE.B.3b, F-IF.C.8a**

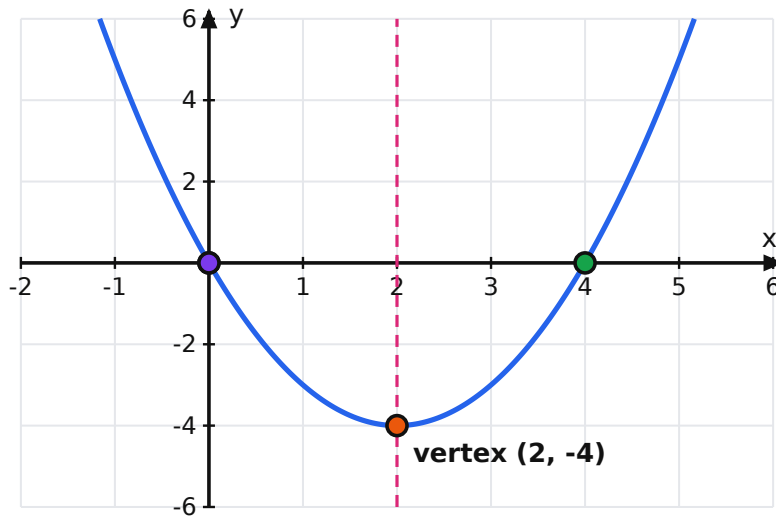
Key facts

- Standard form $ax^2 + bx + c$: y-intercept c .
- Factored form $a(x - r)(x - s)$: zeros r and s .
- Vertex form $a(x - h)^2 + k$: vertex (h, k) and extreme value k .
- Complete the square: add and subtract $(b/2a)^2$.
- $a > 0$ opens up (minimum); $a < 0$ opens down (maximum).

What are the three forms of a quadratic function?

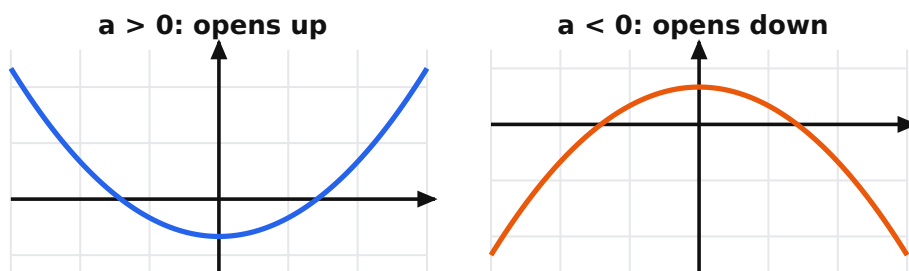
A quadratic function can be written in three equivalent forms. Each form makes a different feature of the parabola easy to read.

Form	Equation	What it shows
Standard	$f(x) = ax^2 + bx + c$	y-intercept c ; axis $x = -\frac{b}{2a}$
Factored	$f(x) = a(x - r)(x - s)$	zeros r and s
Vertex	$f(x) = a(x - h)^2 + k$	vertex (h, k) ; max or min k



$$f(x) = (x - 2)^2 - 4 = x(x - 4): \text{vertex } (2, -4), \text{ zeros } 0 \text{ and } 4, \text{ axis } x = 2.$$

The sign of a gives the direction: positive opens up (the vertex is a minimum) and negative opens down (a maximum). The size of a controls the width: a larger $|a|$ makes a narrower parabola, and a smaller $|a|$ makes a wider one.



Positive a opens up (minimum); negative a opens down (maximum).

How do you complete the square?

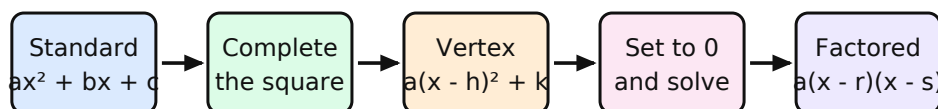
Completing the square rewrites standard form as vertex form. The area model shows the idea: $x^2 + 6x$ is a square of side x plus two strips of area $3x$. Adding the corner 9 completes a square of side $x + 3$.

	x	3
x	x^2	$3x$
3	$3x$	9

$x^2 + 6x + 9 = (x + 3)^2$: the missing corner is 9.

1. If $a \neq 1$, factor a out of the x^2 and x terms.
2. Take half of the x -coefficient and square it.
3. Add and subtract that square, and account for any factor outside.
4. Write the perfect square trinomial as a square and combine constants.

Example: $x^2 - 8x + 5$. Half of -8 is -4 , and $(-4)^2 = 16$. So $x^2 - 8x + 16 - 16 + 5 = (x - 4)^2 - 11$. The vertex is $(4, -11)$.



Each conversion reveals a different feature.

The forms are equal: for instance $(x - 2)^2 - 4 = x^2 - 4x = x(x - 4)$. Choose the form that answers the question: zeros, extreme value, or intercept.

Questions about forms of quadratic functions

What is vertex form of a quadratic?

Vertex form is $f(x) = a(x - h)^2 + k$. The vertex is (h, k) , the axis of symmetry is $x = h$, and k is the maximum when a is negative or the minimum when a is positive.

How do you complete the square?

Take half of the x -coefficient, square it, and add and subtract it. If a is not 1, factor a out first. The result is a perfect square plus a constant.

How do you find the vertex from standard form?

Use $x = -b/(2a)$ for the x-coordinate and substitute into the function for the y-coordinate. Or complete the square to get vertex form.

Which form shows the zeros?

Factored form $a(x - r)(x - s)$ shows the zeros r and s directly, because each factor equal to zero gives a zero.

How can you tell if a parabola has a maximum or minimum?

Look at the sign of a . If a is positive the parabola opens up and has a minimum at the vertex. If a is negative it opens down and has a maximum.