

Algebra 2 - Forms of Quadratic Functions - Answer Key

Common Core A-SSE.B.3a, A-SSE.B.3b, F-IF.C.8a · maths-pdf.fr/en/

Every worksheet problem on the forms of quadratic functions is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where a sign error when finding the vertex or the number to complete the square can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **A-SSE.B.3a, A-SSE.B.3b, F-IF.C.8a**

Exercise 1 - Read the y-intercept

The y-intercept is $f(0)$, which is the constant term c .

$(0,7)$

Exercise 2 - Zeros and vertex from factored form

Zeros: $x=3$ and $x=-1$.

The axis is halfway between them: $x = \frac{3+(-1)}{2} = 1$.

$$f(1) = 2(-2)(2) = -8.$$

Zeros 3 and -1; axis $x = 1$; vertex $(1, -8)$

Exercise 3 - Read vertex form

Compare with $a(x-h)^2+k$: $h=-2$, $k=5$, $a=-1$.

Vertex $(-2, 5)$; maximum value 5 because $a < 0$

Exercise 4 - Vertex from standard form

$$x = -\frac{-8}{2(1)} = 4. \quad f(4) = 16 - 32 + 11 = -5.$$

$(4, -5)$

Exercise 5 - Factor to find zeros

Find two numbers with product -14 and sum -5 : -7 and 2 .

$$(x-7)(x+2)=0.$$

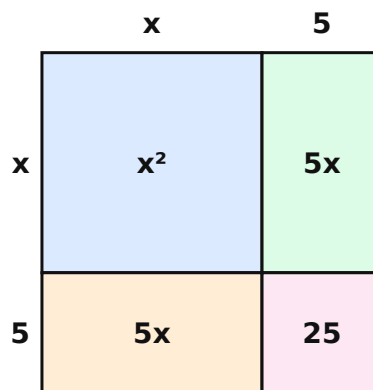
$$x=7, -2$$

Exercise 6 - Factor with a leading coefficient

$$3x^2+10x-8=(3x-2)(x+4). \text{ Check: } 3x^2+12x-2x-8.$$

$$x=\frac{2}{3}, -4$$

Exercise 7 - Complete the square, area model



Corner 25.

Half of 10 is 5, and $5^2=25$.

$$x^2+10x+25=(x+5)^2$$

Exercise 8 - Complete the square

1. Take half of the x -coefficient -10 : -5 . Square it: 25 .
2. Add and subtract 25: $x^2-10x+3$ becomes $(x^2-10x+25)+3-25$.
3. Write the perfect square and combine constants: $y=(x-5)^2-22$.

$$(x-5)^2-22$$

Exercise 9 - Complete the square, a = 2

1. Factor 2 from the x^2 and x terms: $2(x^2+6x)+7$.
2. Take half of the x -coefficient 6: 3. Square it: 9.
3. Add and subtract 9 inside the parentheses: the subtracted amount becomes $2 \cdot 9 = 18$ outside.
4. Write the perfect square and combine constants: $y = 2(x+3)^2 - 11$.

$$2(x+3)^2 - 11; \text{ vertex } (-3, -11)$$

Exercise 10 - Odd coefficient

1. Take half of the x -coefficient 3: $\frac{3}{2}$. Square it: $\frac{9}{4}$.
2. Add and subtract $\frac{9}{4}$: x^2+3x+1 becomes $(x^2+3x+\frac{9}{4})+1-\frac{9}{4}$.
3. Write the perfect square and combine constants: $y = (x+\frac{3}{2})^2 - \frac{5}{4}$.

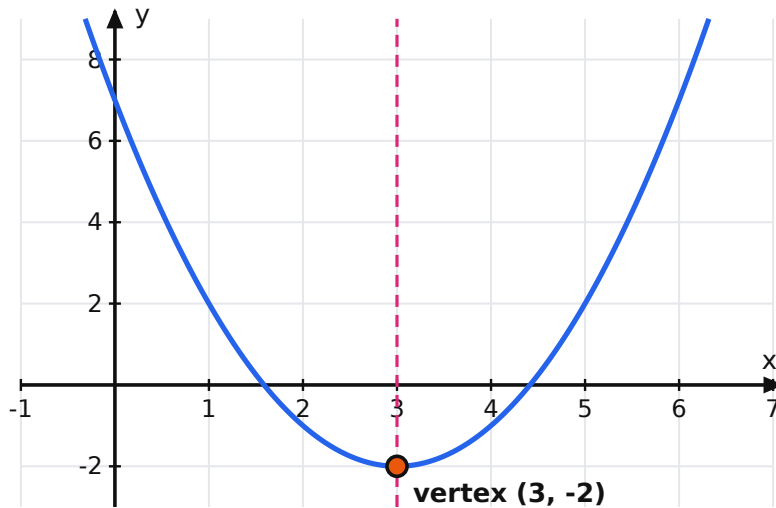
$$(x+\frac{3}{2})^2 - \frac{5}{4}$$

Exercise 11 - Vertex and axis

1. Take half of the x -coefficient 4: 2. Square it: 4.
2. Add and subtract 4: x^2+4x+9 becomes $(x^2+4x+4)+9-4$.
3. Write the perfect square and combine constants: $y = (x+2)^2 + 5$.

Vertex $(-2, 5)$; **axis** $x = -2$; **minimum** 5

Exercise 12 - Equation from the graph, vertex



$$y = (x - 3)^2 - 2.$$

$f(x) = a(x-3)^2 - 2$. Substitute $(5, 2)$: $2 = a(4) - 2$, so $a = 1$.

$$f(x) = (x-3)^2 - 2$$

Exercise 13 - Equation from zeros

$f(x) = a(x+1)(x-5)$. At $x = 0$: $a(1)(-5) = -5$, so $a = 1$.

$$f(x) = (x+1)(x-5)$$

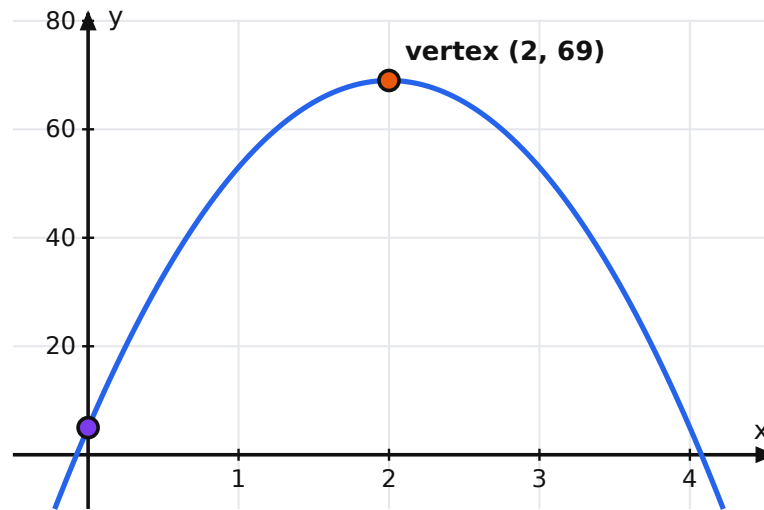
Exercise 14 - Maximum value

$$x = -\frac{8}{2(-2)} = 2. \quad f(2) = -8 + 16 + 1 = 9.$$

Since $a < 0$, this is a maximum. Vertex form: $-2(x-2)^2 + 9$.

Maximum 9 at $x = 2$

Exercise 15 - Thrown ball



Vertex (2, 69).

$$-16(t^2 - 4t) + 5 = -16(t^2 - 4t + 4) + 64 + 5 = -16(t - 2)^2 + 69.$$

Maximum height 69 feet at $t = 2$ seconds

Exercise 16 - Revenue

Vertex form: $-5(p^2 - 20p) = -5(p^2 - 20p + 100) + 500 = -5(p - 10)^2 + 500.$

Factored form: $-5p(p - 20)$, zeros 0 and 20.

Price \$10 gives maximum revenue \$500

Exercise 17 - Fence

Zeros: $w = 0$ and $w = 40$. The vertex is halfway: $w = 20$.

$$A(20) = 20 \cdot 20 = 400.$$

20 ft by 20 ft; area 400 square feet

Exercise 18 - Show two forms are equal

Left: $x^2 - 6x + 9 - 4 = x^2 - 6x + 5.$

Right: $x^2 - 5x - x + 5 = x^2 - 6x + 5.$

Both equal $x^2 - 6x + 5$

Exercise 19 - Which form?

Vertex: $(2, -9)$. Zeros: $(x-2)^2 = 9$, so $x = 2 \pm 3 = 5, -1$.

y-intercept: $f(0) = 4 - 9 = -5$.

Vertex (2, -9); zeros 5 and -1; y-intercept (0, -5)

Exercise 20 - Find the error

$(x+4)^2 = x^2 + 8x + 16$, which adds 16. He must subtract 16: $x^2 + 8x + 3 = (x+4)^2 - 16 + 3$.

$(x+4)^2 - 13$

Exercise 21 - No x-intercepts

A square is never negative, so $(x-2)^2 \geq 0$ and $f(x) \geq 3$. The lowest point is $(2, 3)$, which is above the x-axis, and the parabola opens up.

The minimum value 3 is above 0