

Algebra 2 - Radical and Rational Equations - Lesson

Common Core A-REI.A.2 · maths-pdf.fr/en/

To solve a radical equation, isolate the radical, raise both sides to the matching power, solve, and check every answer. To solve a rational equation, multiply by the least common denominator and discard answers that make a denominator zero. Both methods can create extraneous solutions, so checking is part of the solution.

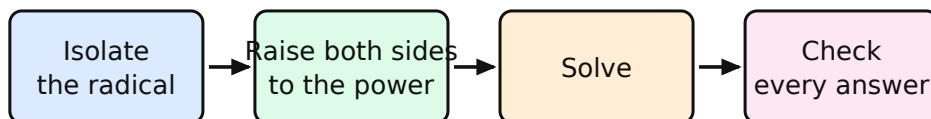
Algebra 2 · Common Core State Standards: **A-REI.A.2**

Key facts

- Isolate the radical before squaring.
- Squaring can create extraneous solutions; always check.
- Rational equations: multiply by the LCD to clear fractions.
- Reject answers that make a denominator 0.
- A principal square root is never negative.

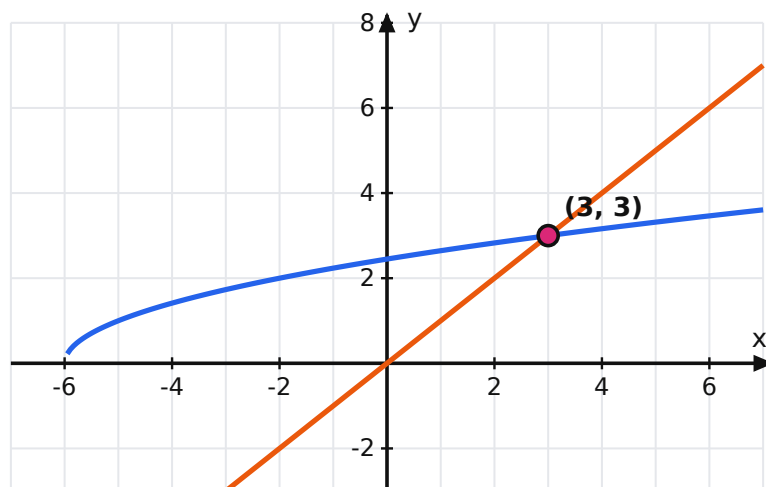
How do you solve a radical equation?

A **radical equation** has the variable under a root. Isolate the radical, raise both sides to the power that removes it, solve, and check. Squaring is not reversible: $a = b$ gives $a^2 = b^2$, but $a^2 = b^2$ also allows $a = -b$. That can create an **extraneous solution**, a value that solves the new equation but not the original one.



Always check: squaring can add extraneous solutions.

Example: $\sqrt{x+6} = x$. Square both sides: $x+6 = x^2$, so $x^2 - x - 6 = 0$ and $(x-3)(x+2) = 0$. Check $x = 3$: $\sqrt{9} = 3$, true. Check $x = -2$: $\sqrt{4} = 2 \neq -2$, false. So $x = -2$ is extraneous.

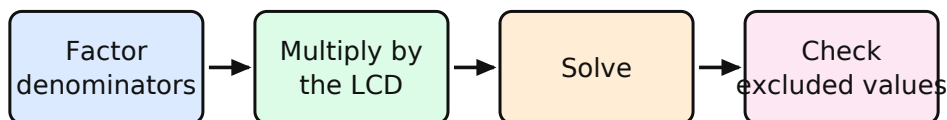


$y = \sqrt{x + 6}$ and $y = x$ meet only at $(3, 3)$: the algebraic solution $x = -2$ is extraneous.

For a cube root, cube both sides. Cube roots do not create extraneous solutions, but check anyway. If two radicals appear, isolate one, square, then isolate the second and square again.

How do you solve a rational equation?

A **rational equation** has the variable in a denominator. Multiply every term by the least common denominator to clear the fractions, solve the resulting equation, and discard any solution that makes a denominator zero.



Reject any solution that makes a denominator zero.

Example: $\frac{2}{x} + \frac{1}{3} = 1$. Multiply by $3x$: $6 + x = 3x$, so $x = 3$. Check: $\frac{2}{3} + \frac{1}{3} = 1$.

Rational equations model work, mixture, and distance problems. If hose A fills a pool in t hours and hose B takes 3 hours longer, together they fill $\frac{1}{t} + \frac{1}{t+3}$ of the pool per hour.

Some equations have no solution at all. For $\sqrt{x} = -3$, no number works because a principal square root is never negative. For $\frac{x}{x-2} = \frac{2}{x-2} + 3$, the only candidate is $x = 2$, and it makes a denominator zero. When every candidate is rejected, write "no solution". Before you start, it helps to state the domain: for $\sqrt{x+6}$ you need $x \geq -6$, and for $\frac{1}{x-2}$ you need $x \neq 2$. These restrictions tell you which candidates to expect to fail.

Equation	Candidate	Check	Result
$\sqrt{x + 6} = x$	$x = -2$	$\sqrt{4} = 2 \neq -2$	extraneous
$\sqrt{x + 6} = x$	$x = 3$	$\sqrt{9} = 3$	valid
$x/(x - 2) = 2/(x - 2) + 3$	$x = 2$	denominator 0	extraneous

Checking candidate solutions.

Questions about radical and rational equations

What is an extraneous solution?

An extraneous solution is a value that solves a transformed equation but not the original. It can appear after squaring both sides or after multiplying by an expression that can be zero.

Why must you check solutions of a radical equation?

Squaring both sides can introduce solutions that make one side the opposite sign. Substitute each answer into the original equation to confirm it works.

How do you solve a rational equation?

Multiply every term by the least common denominator, solve the resulting equation, and reject any solution that makes a denominator equal to zero.

How do you solve an equation with a cube root?

Isolate the cube root and cube both sides. Unlike square roots, a negative number can have a real cube root, so negative answers can be valid.

Can a radical equation have no solution?

Yes. For example, the square root of x equals -3 has no solution because a principal square root is never negative.