

Algebra 2 - Radicals and Rational Exponents - Lesson

Common Core N-RN.A.1, N-RN.A.2 · maths-pdf.fr/en/

A rational exponent is a root and a power at once: a to the m over n equals the n th root of a , raised to the m . The denominator of the exponent is the index of the root and the numerator is the power. Integer exponent properties still hold, and a negative exponent means a reciprocal.

Algebra 2 · Common Core State Standards: **N-RN.A.1, N-RN.A.2**

Key facts

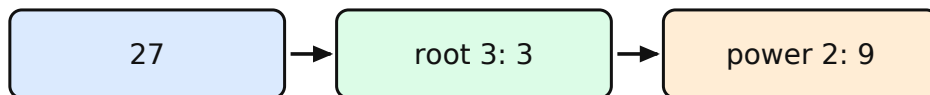
- $a^{(1/n)}$ is the n th root of a .
- $a^{(m/n)} = (\text{nth root of } a)^m$: root index = denominator, power = numerator.
- $a^{(-p)} = 1 / a^p$, also for fractional p .
- Product, quotient and power rules work by adding, subtracting and multiplying exponents.
- Even roots of negative numbers are not real.

What does a rational exponent mean?

Exponent rules for integers extend to fractions. If $(a^n)^m$ rules keep working, then $(a^{1/2})^2 = a^1 = a$, so $a^{1/2}$ must be a number whose square is a . That is the square root.

Rational exponents

For $n \geq 2$ and $a \geq 0$ (any real a when n is odd): $a^{\frac{1}{n}} = \sqrt[n]{a}$. More generally, $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$, and $a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$.

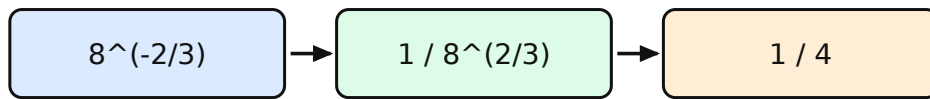


Evaluate $27^{(2/3)}$ in two moves: root first, then power.

The denominator of the exponent is the root index, and the numerator is the power. Taking the root first keeps the numbers small: $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$. You can also take the power first, but then the numbers grow quickly: $27^2 = 729$ and $\sqrt[3]{729} = 9$ gives the same value with more

work.

A negative exponent still means a reciprocal, and the reciprocal is handled first.

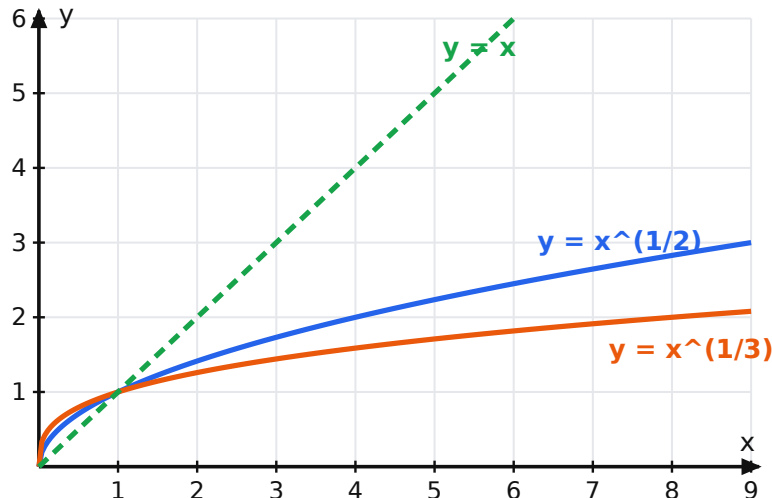


Negative exponent: reciprocal first, then root and power.

Even roots of negative numbers are not real, so the base must be nonnegative when the root index is even. For example, $(-16)^{1/2}$ is not a real number, while $(-27)^{1/3} = -3$ is.

How do the exponent properties work with fractions?

Property	Rule	Example
Product	$a^p \cdot a^q = a^{p+q}$	$x^{1/2} \cdot x^{1/3} = x^{5/6}$
Quotient	$\frac{a^p}{a^q} = a^{p-q}$	$\frac{x^{3/4}}{x^{1/4}} = x^{1/2}$
Power of a power	$(a^p)^q = a^{pq}$	$(x^{2/3})^{3/4} = x^{1/2}$
Power of a product	$(ab)^p = a^p b^p$	$(16x^4)^{1/2} = 4x^2$



Graphs of $y = x^{1/2}$, $y = x^{1/3}$, and $y = x$. Both root functions rise slowly.

How do you simplify an expression with rational exponents?

1. Rewrite radicals as rational exponents.
2. Apply the exponent properties to numbers and variables separately.
3. Rewrite any negative exponent as a reciprocal.
4. Evaluate roots of perfect powers and write the final answer without negative exponents.

Example: $\frac{(8x^6)^{2/3}}{x^{1/2}}$. First $(8x^6)^{2/3} = 8^{2/3} x^4 = 4x^4$. Then divide: $4x^4 \div x^{1/2} = 4x^{7/2}$.

Exponent form	Radical form	Value
$16^{(1/2)}$	square root of 16	4
$8^{(2/3)}$	(cube root of 8) ²	4
$81^{(3/4)}$	(4th root of 81) ³	27
$32^{(-1/5)}$	1 / (5th root of 32)	1/2

Four conversions: the root index is the denominator of the exponent.

Questions about radicals and rational exponents

What does a fractional exponent mean?

The denominator is the index of a root and the numerator is a power. So 16 to the 3/4 means the fourth root of 16, cubed, which equals 8.

Which comes first, the root or the power?

Either order gives the same value, but taking the root first is easier with whole numbers. For 27 to the 2/3, find the cube root 3, then square it to get 9.

How do you handle a negative rational exponent?

Take the reciprocal first, then evaluate. For 8 to the -2/3, write 1 divided by 8 to the 2/3, which is 1 divided by 4.

Can a rational exponent be applied to a negative base?

Only when the root index is odd. The cube root of -8 is -2, but the square root of -8 is not a real number.

Why does x to the 1/2 equal the square root of x ?

The product rule requires x to the 1/2 times x to the 1/2 to equal x to the 1. The only nonnegative number that multiplies by itself to give x is the square root.