

Algebra 2 - Rational Expressions - Lesson

Common Core A-APR.D.7, A-SSE.A.2 · maths-pdf.fr/en/

A rational expression is a quotient of polynomials. To simplify, factor the numerator and denominator and cancel common factors, and state the excluded values that make the denominator zero. Multiply by multiplying across, divide by multiplying by the reciprocal, and add or subtract over a common denominator.

Algebra 2 · Common Core State Standards: **A-APR.D.7, A-SSE.A.2**

Key facts

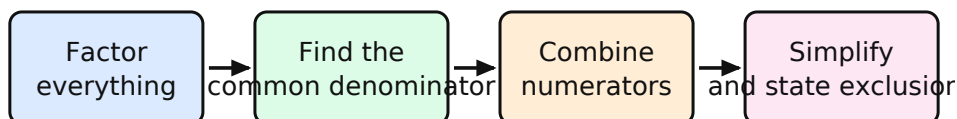
- Rational expression = polynomial $\div$ polynomial.
- Factor first, then cancel common factors only.
- Excluded values make the original denominator 0.
- Divide by multiplying by the reciprocal.
- Add or subtract over the least common denominator.

What is a rational expression?

Rational expression

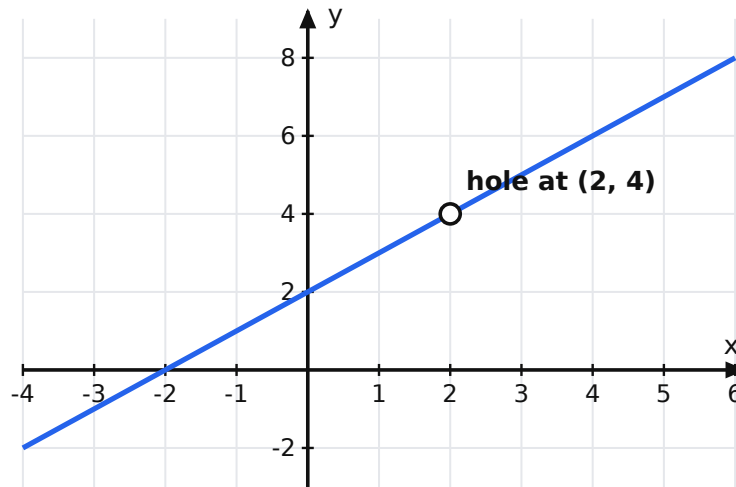
A **rational expression** is a quotient of two polynomials, $\frac{p(x)}{q(x)}$ with $q(x) \neq 0$. Values of x that make the denominator zero are **excluded values**.

Rational expressions behave like fractions. To simplify, factor the numerator and the denominator and cancel common factors. For example, $\frac{x^2-9}{x^2+x-6} = \frac{(x-3)(x+3)}{(x+3)(x-2)} = \frac{x-3}{x-2}$, with $x \neq -3$ and $x \neq 2$.



Work through rational expressions in four steps.

The excluded values come from the original denominator, even after canceling. The simplified graph keeps a hole at each canceled factor.



$y = (x^2 - 4)/(x - 2)$ is the line $y = x + 2$ with a hole at $x = 2$.

How do you multiply and divide rational expressions?

Multiply numerators and denominators, factor first, and cancel. To divide, multiply by the reciprocal of the second expression. Example: $\frac{x^2-1}{x+3} \cdot \frac{x^2+3x}{x+1} = \frac{(x-1)(x+1)}{x+3} \cdot \frac{x(x+3)}{x+1} = x(x-1)$.

How do you add and subtract rational expressions?

Write each expression over a common denominator, then combine the numerators. Find the least common denominator by factoring each denominator.

1. Factor each denominator.
2. Build the LCD from every factor, using the highest power.
3. Rewrite each fraction over the LCD.
4. Add or subtract the numerators and simplify.

Example: $\frac{3}{x+2} + \frac{5}{x-1} = \frac{3(x-1)+5(x+2)}{(x+2)(x-1)} = \frac{8x+7}{(x+2)(x-1)}$.

Fraction of the job done in 1 hour

A	1/6	
D	1/4	
together 5/12		

Aisha paints 1/6 of the fence per hour and Diego 1/4 per hour: together 5/12.

Rational expressions model rates. If Aisha paints a fence in 6 hours and Diego paints it in 4 hours, their combined rate is $\frac{1}{6} + \frac{1}{4} = \frac{5}{12}$ of the fence per hour, so together they need $\frac{12}{5} = 2.4$

hours. Recognizing structure also helps: $x^4 - 16 = (x^2)^2 - 4^2 = (x^2 - 4)(x^2 + 4)$.

Expression	Excluded values	Simplified form
$(x^2 - 9)/(x^2 + x - 6)$	-3 and 2	$(x - 3)/(x - 2)$
$(x^2 - 4)/(x - 2)$	2	$x + 2$
$6x/(3x^2)$	0	$2/x$

Excluded values come from the original denominator.

Questions about rational expressions

How do you simplify a rational expression?

Factor the numerator and the denominator completely, then cancel common factors. State the excluded values from the original denominator.

What are excluded values?

Excluded values are the inputs that make a denominator zero. They stay excluded even if the factor cancels.

How do you add rational expressions with different denominators?

Factor the denominators, find the least common denominator, rewrite each expression over it, and add the numerators.

How do you divide rational expressions?

Multiply by the reciprocal of the divisor, then factor and cancel. For example, a/b divided by c/d equals a/b times d/c .

Can you cancel terms that are added?

No. You can cancel only common factors. In $(x + 3)/3$, you cannot cancel the 3s because 3 is a term in the numerator, not a factor.