

Algebra 2 - Arithmetic and Geometric Sequences and Series - Lesson

Common Core F-BF.A.2, A-SSE.B.4, F-IF.A.3 · maths-pdf.fr/en/

A sequence is a function on the positive integers. An arithmetic sequence adds a common difference d and has $a_n = a_1 + (n - 1)d$, while a geometric sequence multiplies by a common ratio r and has $a_n = a_1 \cdot r^{(n-1)}$. The sum of the first n terms is called a series, and each type has a short sum formula.

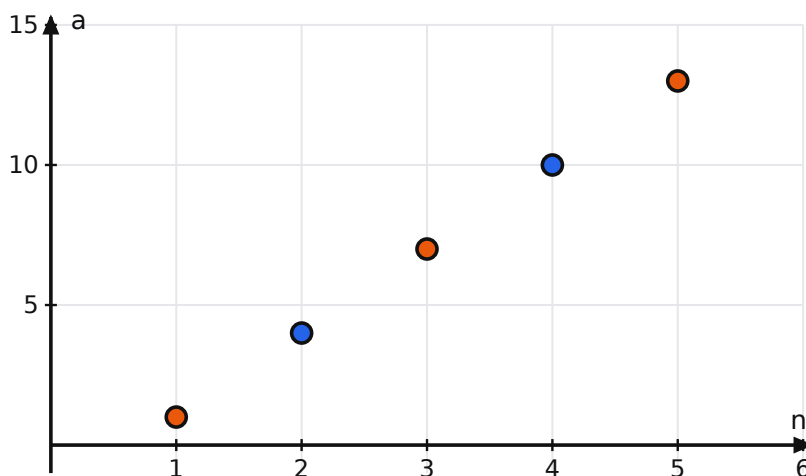
Algebra 2 · Common Core State Standards: **F-BF.A.2, A-SSE.B.4, F-IF.A.3**

Key facts

- A sequence is a function whose domain is the positive integers.
- Arithmetic: $a_n = a_1 + (n - 1)d$; geometric: $a_n = a_1 \cdot r^{(n-1)}$.
- Recursive formulas build each term from the previous one.
- Arithmetic sum: $S_n = n(a_1 + a_n)/2$.
- Geometric sum: $S_n = a_1(1 - r^n)/(1 - r)$, $r \neq 1$.

What is a sequence?

A **sequence** is a function whose domain is the positive integers: $a_1, a_2, a_3, \dots$ written here as a list of terms a_n . A sequence can be given by an **explicit formula** such as $a_n = 3n - 2$, or by a **recursive formula** that tells how to get each term from the previous ones, such as $a_1 = 5$ and $a_n = a_{n-1} + 4$.

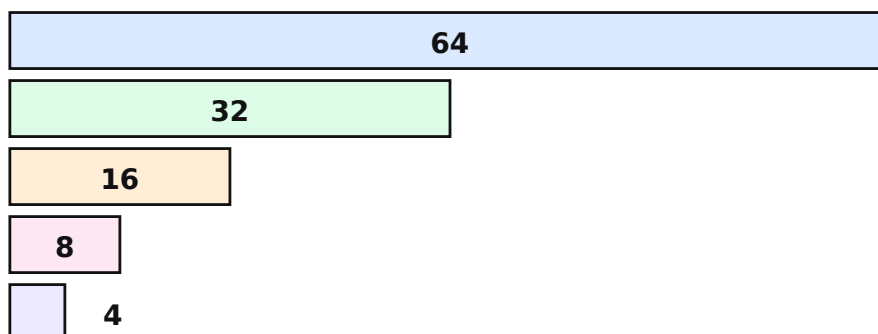


The first five terms of $a_n = 3n - 2$ lie on a line.

Arithmetic and geometric sequences

In an **arithmetic sequence** each term adds a common difference d : $a_n = a_1 + (n-1)d$. In a **geometric sequence** each term multiplies by a common ratio r : $a_n = a_1 r^{n-1}$.

Arithmetic sequences are linear and geometric sequences are exponential. For 7, 11, 15, 19, ..., $d = 4$, so $a_n = 7 + (n-1)4 = 4n + 3$. For 80, 40, 20, 10, ..., $r = \frac{1}{2}$, so $a_n = 80\left(\frac{1}{2}\right)^{n-1}$.



A geometric sequence with ratio 1/2: each bar is half the one above.

How do you add the terms of a series?

A **series** is the sum of the terms of a sequence. Let S_n be the sum of the first n terms. For an arithmetic series, pair the first and last terms: $S_n = \frac{n(a_1 + a_n)}{2}$. For a geometric series with $r \neq 1$: $S_n = a_1 \cdot \frac{1-r^n}{1-r}$.

1	2	3	4	5	6	7	8	9	10
10	9	8	7	6	5	4	3	2	1
11	11	11	11	11	11	11	11	11	11

10 pairs of 11 = 110, so $1 + 2 + \dots + 10 = 55$

Pairing the sequence with its reverse: every column sums to 11.

1. Decide whether the sequence is arithmetic or geometric.
2. Find a_1 , the number of terms n , and d or r .
3. Choose the right sum formula and substitute.

Example: $3 + 6 + 12 + \dots$ with 8 terms has $a_1 = 3$, $r = 2$, so $S_8 = 3 \cdot \frac{1-2^8}{1-2} = 3 \cdot 255 = 765$. A geometric series is the model for savings plans: if you deposit \$200 every year at 5%, the balance after 10 years is a geometric series.

Type	Rule	n-th term	Sum of n terms
Arithmetic	add d	$a_1 + (n - 1)d$	$n(a_1 + a_n)/2$
Geometric	multiply by r	$a_1 \cdot r^{(n - 1)}$	$a_1(1 - r^n)/(1 - r)$

Arithmetic and geometric sequences side by side.

Questions about arithmetic and geometric sequences and series

What is the difference between an arithmetic and a geometric sequence?

An arithmetic sequence adds the same number each step, like 3, 7, 11. A geometric sequence multiplies by the same number each step, like 3, 6, 12.

What is the explicit formula for an arithmetic sequence?

$a_n = a_1 + (n - 1)d$, where a_1 is the first term and d is the common difference.

What is the sum formula for a geometric series?

$S_n = a_1 (1 - r^n) / (1 - r)$ for r not equal to 1, where a_1 is the first term, r is the common ratio, and n is the number of terms.

What is a recursive formula?

A recursive formula gives the first term and a rule that builds each term from the previous one, for example $a_1 = 2$ and $a_n = 3 a_{(n-1)}$.

Why is a sequence a function?

Each position number n has exactly one term a_n . The domain is the positive integers, so a sequence is a function whose graph is a set of separate points.