

Algebra 2 - Arithmetic and Geometric Sequences and Series - Answer Key

Common Core F-BF.A.2, A-SSE.B.4, F-IF.A.3 · maths-pdf.fr/en/

Every worksheet problem on arithmetic and geometric sequences and series is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where mixing up the arithmetic and geometric formulas can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **F-BF.A.2, A-SSE.B.4, F-IF.A.3**

Exercise 1 - List the terms

Substitute $n=1,2,3,4,5$: 1,4,7,10,13.

1, 4, 7, 10, 13

Exercise 2 - Recursive formula

Start at 5 and add 4 each time.

5, 9, 13, 17, 21

Exercise 3 - Arithmetic or geometric?

Sequence	Differences / ratios	Type
3, 6, 12, 24	ratio 2	geometric
20, 17, 14, 11	difference -3	arithmetic
2, 6, 18, 54	ratio 3	geometric

Completed table.

$6 \div 3 = 2$, $12 \div 6 = 2$: geometric. $17 - 20 = -3$, $14 - 17 = -3$: arithmetic. Ratios 3, 3, 3: geometric.

Geometric, arithmetic, geometric

Exercise 4 - Explicit arithmetic formula

$$d = 4, a_1 = 7: a_n = 7 + (n-1)4 = 4n + 3.$$

$$a_{20} = 4(20) + 3 = 83.$$

$$a_n = 4n + 3; a_{20} = 83$$

Exercise 5 - Find a term

$$a_{15} = -4 + 14(3) = -4 + 42 = 38.$$

38

Exercise 6 - Find the position

$$a_n = 2 + (n-1)3 = 3n - 1. \text{ Set } 3n - 1 = 62: n = 21.$$

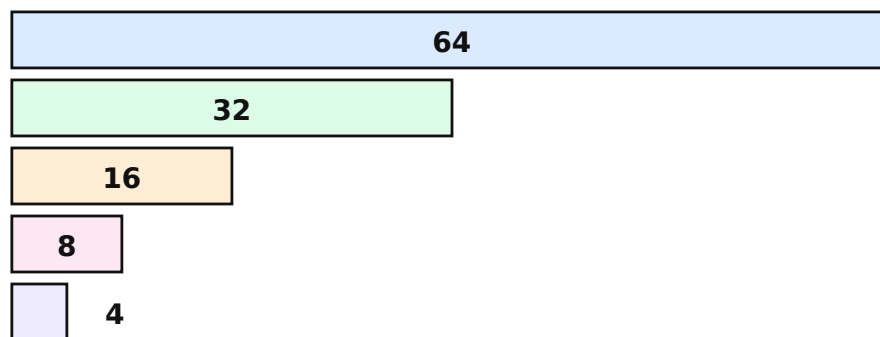
The 21st term

Exercise 7 - Geometric formula

$$r = 2: a_n = 3 \cdot 2^{n-1}. a_8 = 3 \cdot 2^7 = 3 \cdot 128 = 384.$$

$$a_8 = 384$$

Exercise 8 - Geometric with a fraction



A geometric sequence with ratio $1/2$: each bar is half the one above.

$$a_6 = 80 \left(\frac{1}{2} \right)^5 = \frac{80}{32} = 2.5.$$

2.5

Exercise 9 - Recursive geometric

Multiply by 3 each time: 2, 6, 18, 54.

2, 6, 18, 54

Exercise 10 - Gauss's sum

1	2	3	4	5	6	7	8	9	10
10	9	8	7	6	5	4	3	2	1
11	11	11	11	11	11	11	11	11	11

10 pairs of 11 = 110, so $1 + 2 + \dots + 10 = 55$

Pairing the sequence with its reverse: every column sums to 11.

$$S_{100} = \frac{100(1+100)}{2} = 50 \cdot 101.$$

5,050

Exercise 11 - Arithmetic series

$$a_{20} = 5 + 19(3) = 62.$$

$$S_{20} = \frac{20(5+62)}{2} = 10 \cdot 67 = 670.$$

670

Exercise 12 - Theater seats

$$a_{15} = 12 + 14(2) = 40.$$

$$S_{15} = \frac{15(12+40)}{2} = 15 \cdot 26 = 390.$$

390 seats

Exercise 13 - Geometric series

$$S_8 = 3 \cdot \frac{1-2^8}{1-2} = 3 \cdot \frac{-255}{-1} = 765.$$

765

Exercise 14 - Halving series

$$r = \frac{1}{2}. S_7 = 64 \cdot \frac{1-(1/2)^7}{1-1/2} = 64 \cdot \frac{127/128}{1/2} = 127.$$

127

Exercise 15 - Doubling pennies

$$S_{30} = 1 \cdot \frac{1-2^{30}}{1-2} = 2^{30} - 1 = 1,073,741,823 \text{ cents.}$$

Divide by 100 to get dollars.

\$10,737,418.23

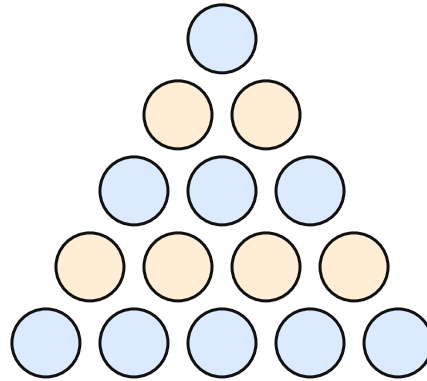
Exercise 16 - Savings plan

Geometric series with $a_1 = 200$, $r = 1.05$, $n = 10$.

$$S_{10} = 200 \cdot \frac{1-1.05^{10}}{1-1.05} = 200 \cdot \frac{1.62889-1}{0.05} \approx 2,515.58.$$

About \$2,515.58

Exercise 17 - Triangular stack



$$5 + 4 + \dots + 1 = 15 \text{ logs}$$

A stack of logs with 5 in the bottom row.

The sum $1+2+\dots+20$ has $n=20$, $a_1=1$, $a_{20}=20$.

$$S_{20} = \frac{20(1+20)}{2} = 210.$$

210 logs

Exercise 18 - Fibonacci

Add the two previous terms: 1, 1, 2, 3, 5, 8, 13, 21.

The differences 0, 1, 1, 2 and the ratios 1, 2, 1.5 are not constant.

1, 1, 2, 3, 5, 8, 13, 21; neither

Exercise 19 - Find the error

The differences are 2, 4, 8, which are not constant. The ratios are all 2, so the sequence is geometric.

It is geometric with $r = 2$

Exercise 20 - Two terms given

$$a_8 - a_3 = 5d = 25, \text{ so } d = 5.$$

$$a_1 = 11 - 2(5) = 1. \quad a_n = 1 + 5(n-1) = 5n - 4.$$

$$d = 5, \quad a_1 = 1, \quad a_n = 5n - 4$$