

Algebra 2 - Transformations of Functions - Lesson

Common Core F-BF.B.3, F-IF.C.7 · maths-pdf.fr/en/

A transformation of $f(x)$ shifts, stretches, compresses, or reflects its graph. Adding k outside moves the graph up or down, while $f(x - h)$ moves it right by h . A factor a stretches it vertically and $f(bx)$ compresses it horizontally. A function is even if $f(-x) = f(x)$ and odd if $f(-x) = -f(x)$.

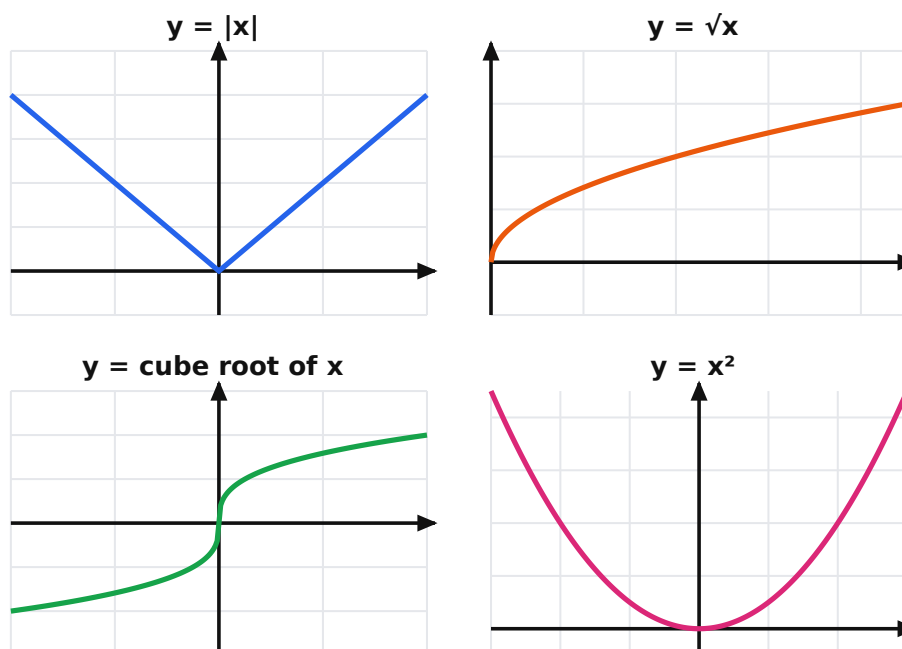
Algebra 2 · Common Core State Standards: **F-BF.B.3, F-IF.C.7**

Key facts

- $f(x) + k$: up k ; $f(x - h)$: right h .
- $a \cdot f(x)$: vertical stretch by $|a|$; negative a reflects over the x -axis.
- $f(bx)$: horizontal compression by $1/|b|$; negative b reflects over the y -axis.
- Even: $f(-x) = f(x)$; odd: $f(-x) = -f(x)$.
- Piecewise functions use different rules on different intervals.

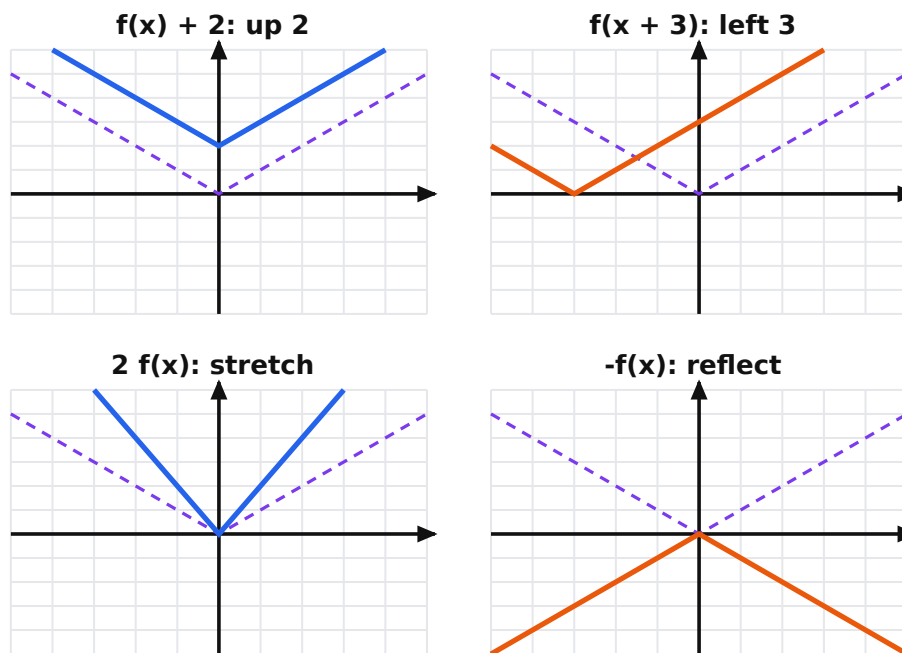
What do transformations do to a graph?

A **transformation** changes a parent function $f(x)$ into a new function by moving, stretching, or flipping its graph. Learn the parent graphs first: $|x|$, $\sqrt{x}$, $\sqrt[3]{x}$, and x^2 .



Transformation rules

For $k > 0$: $f(x) + k$ shifts up k ; $f(x) - k$ shifts down. $f(x - h)$ shifts right h ; $f(x + h)$ shifts left h .
 $a \cdot f(x)$ stretches vertically by $|a|$ (and flips over the x-axis if $a < 0$). $f(bx)$ compresses horizontally by $\frac{1}{|b|}$ (and flips over the y-axis if $b < 0$).

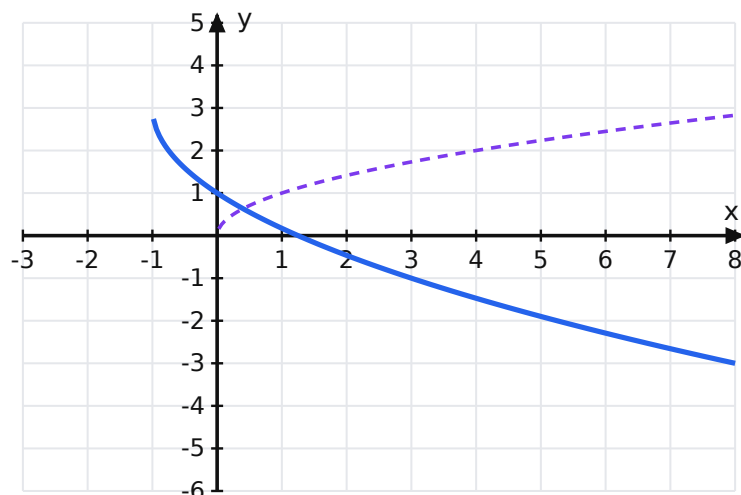


Dashed: the parent $f(x) = |x|$. Solid: the transformed graph.

Changes to the output ($f(x) + k$, $a f(x)$) behave as you expect. Changes inside the function ($f(x + h)$, $f(bx)$) work backwards. For example, $f(x + 3)$ moves left because the input must be 3 smaller to give the same output.

How do you describe several transformations at once?

Take $g(x) = -2\sqrt{x+1} + 3$, built from $f(x) = \sqrt{x}$. Read it from the inside out: shift left 1, stretch vertically by 2, reflect over the x-axis, shift up 3. Each point (x, y) on f moves to $(x - 1, -2y + 3)$. The domain is $x \geq -1$ and the range is $y \leq 3$.



$g(x) = -2\sqrt{x+1} + 3$ starts at $(-1, 3)$ and falls. The dashed curve is $y = \sqrt{x}$.

Reflections and stretches combine. If $a < 0$, the graph is flipped over the x -axis and stretched by $|a|$. If $b < 0$ in $f(bx)$, the graph is flipped over the y -axis. Always check a transformation on one or two points, such as the vertex of an absolute value graph or the starting point of a square root graph.

To graph a transformed function, start with a few key points of the parent graph and move each one. For $f(x) = |x|$ use $(-2, 2)$, $(0, 0)$, and $(2, 2)$. For $g(x) = |x-3|+1$ these points move right 3 and up 1 to $(1, 3)$, $(3, 1)$, and $(5, 3)$. Plot the three new points, then draw the same V shape through them. This point-mapping method also works for square root and cube root graphs, and it lets you verify the algebra against the picture.

What are even and odd functions?

A function is **even** if $f(-x) = f(x)$ (symmetric about the y -axis), like $x^4 - 3x^2$. It is **odd** if $f(-x) = -f(x)$ (symmetric about the origin), like $x^3 + x$. Most functions are neither. A **piecewise function** uses different rules on different intervals, such as $f(x) = x+2$ for $x < 1$ and $f(x) = x^2$ for $x \geq 1$.

Transformation	Effect on (x, y)
$f(x) + k$	$(x, y + k)$
$f(x - h)$	$(x + h, y)$
$a \cdot f(x)$	$(x, a \cdot y)$
$f(bx)$	$(x / b, y)$
$-f(x)$	$(x, -y)$

How each transformation moves a point of the parent graph.

Questions about transformations of functions

What does $f(x + 3)$ do to the graph?

It shifts the graph 3 units to the left. Adding inside the function moves the graph the opposite way from what the sign suggests.

What is the difference between a stretch and a compression?

A vertical stretch multiplies outputs by a number greater than 1. A horizontal compression, $f(bx)$ with b greater than 1, squeezes the graph toward the y-axis.

How do you know if a function is even or odd?

Compare $f(-x)$ with $f(x)$. If they are equal the function is even. If $f(-x) = -f(x)$ it is odd. Otherwise it is neither.

In what order should you apply transformations?

Work inside out like order of operations: horizontal changes first, then stretches and reflections, then vertical shifts.

What are the parent functions in Algebra 2?

The key parent functions are the absolute value, square root, cube root, quadratic, cubic, exponential and logarithm functions.