

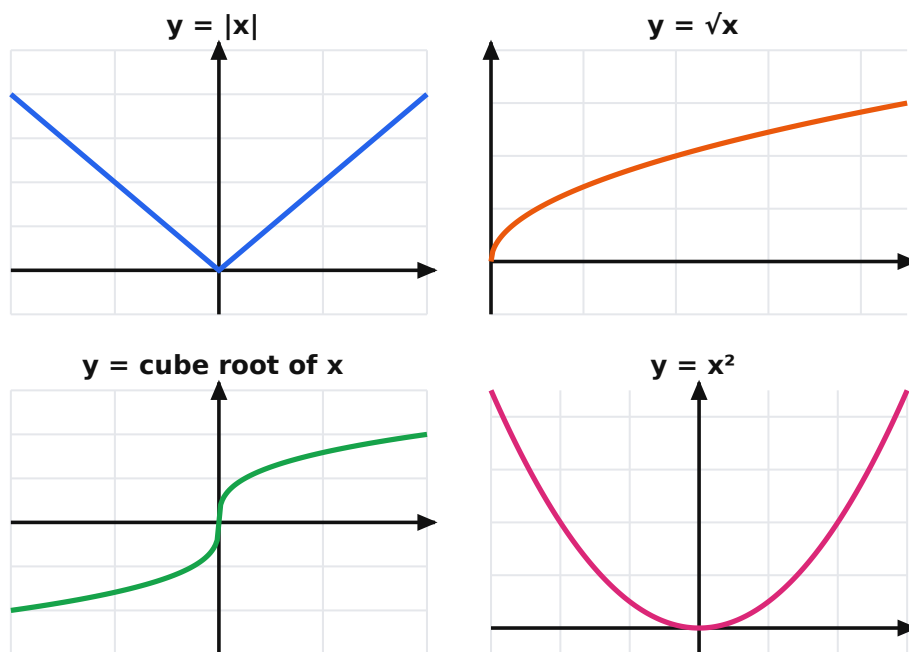
Algebra 2 - Transformations of Functions - Answer Key

Common Core F-BF.B.3, F-IF.C.7 · maths-pdf.fr/en/

Every worksheet problem on transformations of functions is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where moving the graph the wrong way for $f(x + h)$ can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **F-BF.B.3, F-IF.C.7**

Exercise 1 - Name the parent function



Four parent functions.

Left to right, top to bottom: absolute value (vertex at the origin), square root (starting point at the origin), cube root (center at the origin), and quadratic (vertex at the origin).

Absolute value, square root, cube root, quadratic

Exercise 2 - Shift an absolute value

$x+4$ shifts the graph left 4 and -2 shifts it down 2.

Left 4, down 2; vertex (-4, -2)

Exercise 3 - Write an equation from a description

Right 3: replace x with $x-3$. Up 5: add 5.

The radicand must be nonnegative: $x-3 \geq 0$.

$$g(x) = \sqrt{x-3} + 5, \text{ domain } x \geq 3$$

Exercise 4 - Two reflections

$-f(x)$ reflects over the x-axis: the graph goes down-right from the origin.

$f(-x)$ reflects over the y-axis: the graph goes up-left, with domain $x \leq 0$.

g: reflection over the x-axis; h: reflection over the y-axis

Exercise 5 - Vertical stretch

Every output is multiplied by 3, a vertical stretch by a factor of 3.

$$g(1) = 3(1) = 3; g(-2) = 3(2) = 6.$$

Stretch by 3; $g(1) = 3$, $g(-2) = 6$

Exercise 6 - Compression and stretch

$f(2x) = (2x)^2 = 4x^2 = 4f(x)$. For a quadratic, a horizontal compression by $\frac{1}{2}$ looks the same as a vertical stretch by 4.

$$f(2x) = 4x^2 = 4f(x)$$

Exercise 7 - Transform a table

x	-1	1	2	4
g(x)	6	3	0	7

Each point moves right 1 and up 2.

Each point (x,y) moves to $(x+1,y+2)$: $(-2,4)$ becomes $(-1,6)$ is written as the point $(-1,6)$; $(0,1)$ gives $(1,3)$; $(1,-2)$ gives $(2,0)$; $(3,5)$ gives $(4,7)$.

g: $(-1, 6), (1, 3), (2, 0), (4, 7)$

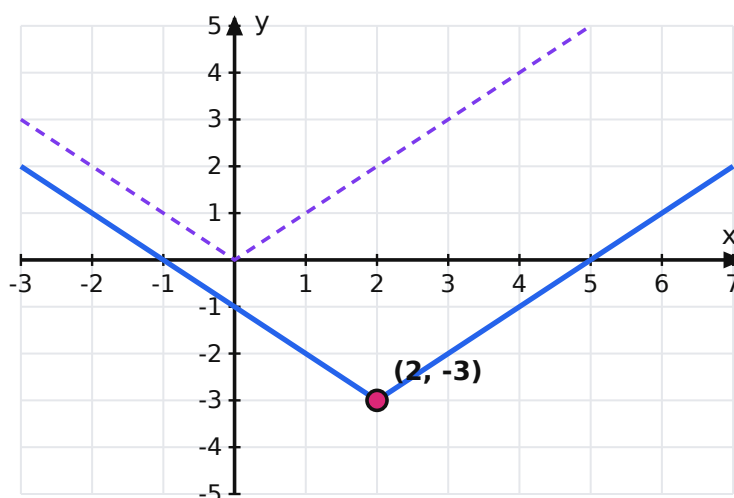
Exercise 8 - Reflect a table

$-f(x)$ changes the sign of y : $(-2,-4), (0,-1), (1,2), (3,-5)$.

$f(-x)$ changes the sign of x : $(2,4), (0,1), (-1,-2), (-3,5)$.

h reflects over the x-axis; k reflects over the y-axis

Exercise 9 - Write from a graph



Vertex at $(2, -3)$, same slope as $|x|$.

The vertex moved from $(0,0)$ to $(2,-3)$: right 2, down 3. The slopes are still ± 1 .

$$g(x) = |x-2|-3$$

Exercise 10 - Several transformations

Left 1, vertical stretch by 2, reflection over the x-axis, up 3.

$$g(3) = -2\sqrt{4} + 3 = -4 + 3 = -1.$$

Domain: $x+1 \geq 0$, so $x \geq -1$. Since $\sqrt{x+1} \geq 0$, $g(x) \leq 3$.

$g(3) = -1$; domain $x \geq -1$; range $y \leq 3$

Exercise 11 - Cube root

The center moves from (0,0) to (1,-2).

$$g(9) = \sqrt[3]{8} - 2 = 2 - 2 = 0.$$

Center (1, -2); $g(9) = 0$

Exercise 12 - Even function

$$f(-x) = (-x)^4 - 3(-x)^2 = x^4 - 3x^2 = f(x).$$

Even: symmetric about the y-axis

Exercise 13 - Odd function

$$g(-x) = (-x)^3 + (-x) = -x^3 - x = -(x^3 + x) = -g(x).$$

Odd: symmetric about the origin

Exercise 14 - Neither

$$h(1) = 2 \text{ and } h(-1) = 1 - 1 = 0.$$

$$h(-1) \neq h(1) \text{ (not even) and } h(-1) \neq -h(1) \text{ (not odd).}$$

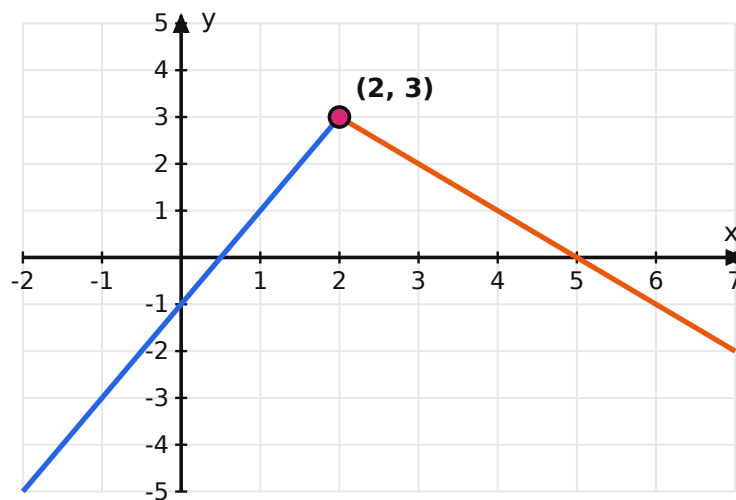
Neither

Exercise 15 - Evaluate a piecewise function

$-3 < 1$: use $x+2$, so $f(-3) = -1$. $1 \geq 1$: use x^2 , so $f(1) = 1$. $f(4) = 16$.

-1, 1, 16

Exercise 16 - Graph a piecewise function



Both pieces meet at $(2, 3)$.

$f(0) = -1$; $f(2) = 2(2) - 1 = 3$; $f(4) = 5 - 4 = 1$.

Check the other piece at $x = 2$: $5 - 2 = 3$, so the pieces meet.

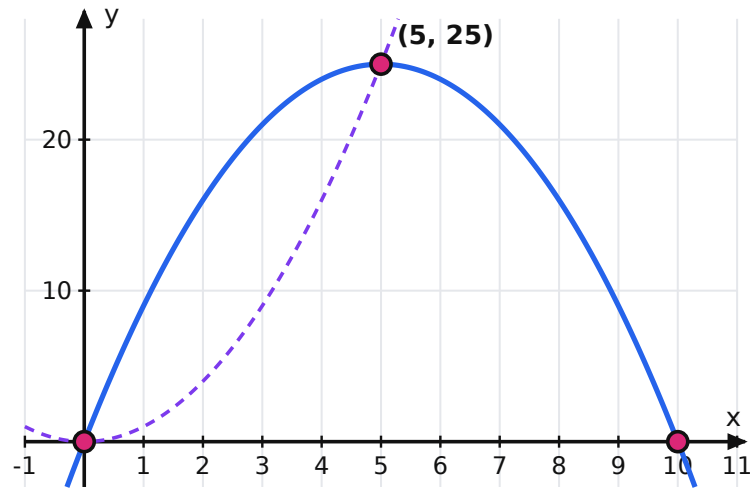
$f(0) = -1$, $f(2) = 3$, $f(4) = 1$; no break

Exercise 17 - Temperature deviation

The graph of $|t|$ is shifted right 68. $D(75) = |7| = 7$; $D(60) = |-8| = 8$.

Right 68; $D(75) = 7$ and $D(60) = 8$

Exercise 18 - Bridge arch



Zeros at $x = 0$ and $x = 10$.

Reflect over the x-axis, shift right 5, shift up 25. The vertex $(5, 25)$ is the maximum.

Ground: $-(x-5)^2 + 25 = 0$, so $(x-5)^2 = 25$ and $x = 0$ or 10 .

Maximum 25 ft; the arch is 10 ft wide at the ground

Exercise 19 - Horizontal stretch

For g : $\frac{x}{2} = 2$ gives $x = 4$, so $(4, 3)$. For h : $2x = 2$ gives $x = 1$, so $(1, 3)$.

g : $(4, 3)$; h : $(1, 3)$

Exercise 20 - Find the error

$f(x+3)$ gives the same output 3 units earlier. If $f(0) = 5$, then $g(-3) = f(0) = 5$, so the point moves from $x = 0$ to $x = -3$.

The graph shifts left 3

Exercise 21 - Inside and outside

Solve $2x - 4 = 6$: $x = 5$. The output is unchanged.

$(5, 5)$