

Algebra 2 - Graphs of Trigonometric Functions and Identities - Answer Key

Common Core F-TF.B.5, F-TF.C.8, F-IF.C.7e · maths-pdf.fr/en/

Every worksheet problem on graphs of trigonometric functions and the Pythagorean identity is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where using B instead of $2\pi/B$ for the period can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **F-TF.B.5, F-TF.C.8, F-IF.C.7e**

Exercise 1 - Read the parameters

Amplitude $|3| = 3$. Period $\frac{2\pi}{2} = \pi$. Midline $y = 1$.

Amplitude 3; period π ; midline $y = 1$

Exercise 2 - Reflection and stretch

Amplitude $|-2| = 2$. $B = \frac{1}{2}$, so the period is $\frac{2\pi}{1/2} = 4\pi$.

The negative sign reflects the graph over the x-axis, so it starts at its minimum.

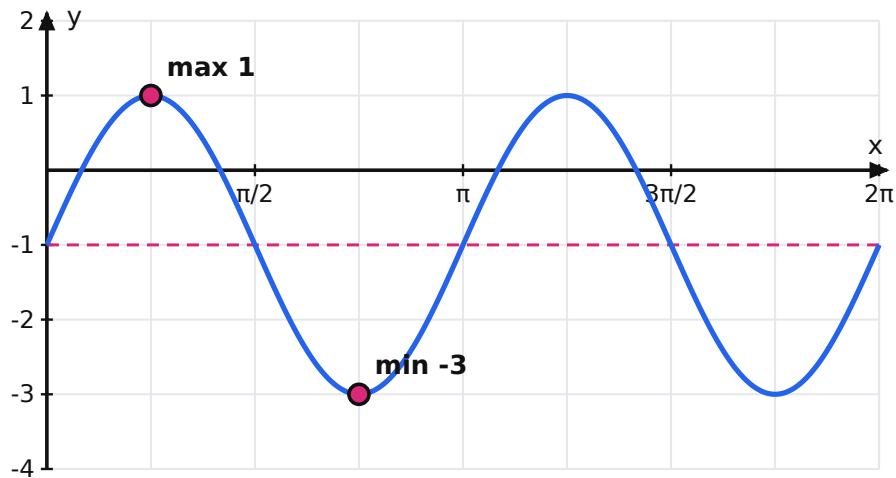
Amplitude 2; period 4π ; reflected

Exercise 3 - Phase shift

The form $x - C$ with $C = \frac{\pi}{3}$ shifts the graph right. The midline is $y = 2$.

Right $\pi/3$; midline $y = 2$

Exercise 4 - Equation from a graph



Amplitude 2, period π .

Amplitude $\frac{1-(-3)}{2}=2$. Midline $D=-1$. $B=\frac{2\pi}{\pi}=2$.

$$y = 2\sin(2x) - 1$$

Exercise 5 - Write from features

$$A=4, B=\frac{2\pi}{\pi}=2, D=2.$$

$$y = 4\sin(2x) + 2$$

Exercise 6 - Maximum and minimum

$\cos x$ ranges from -1 to 1 , so $5\cos x - 3$ ranges from $-5 - 3 = -8$ to $5 - 3 = 2$.

Maximum 2; minimum -8

Exercise 7 - Ferris wheel

$$h(0) = 25 - 20 = 5. \quad h(15) = 25 - 20\cos\pi = 45.$$

$$h(10) = 25 - 20\cos\frac{2\pi}{3} = 25 + 10 = 35.$$

$$\text{Period: } \frac{2\pi}{\pi/15} = 30 \text{ seconds.}$$

5 ft, 45 ft, 35 ft; period 30 s

Exercise 8 - Tides

Amplitude 4. $B = \frac{\pi}{6}$, so the period is $\frac{2\pi}{\pi/6} = 12$ hours.

$$y(3) = 4\sin\frac{\pi}{2} + 6 = 10.$$

Amplitude 4 ft; period 12 hours; 10 ft at 3 AM

Exercise 9 - Temperature

Midline 65, amplitude 15: maximum 80, minimum 50.

$$T(15) = 15\sin\frac{\pi}{2} + 65 = 80: \text{ the high occurs at 3 PM.}$$

High 80°F, low 50°F; T(15) = 80°F

Exercise 10 - Find B from the period

$$\frac{2\pi}{B} = \frac{\pi}{2}, \text{ so } B = 4.$$

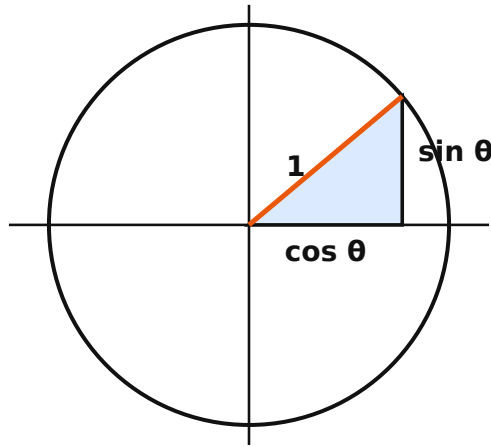
$$B = 4$$

Exercise 11 - Identity at an angle

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1.$$

Verified

Exercise 12 - Find cosine in quadrant II



The right triangle in the unit circle: $\cos^2 \theta + \sin^2 \theta = 1$.

$$\cos^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}. \text{ In quadrant II cosine is negative: } \cos \theta = -\frac{4}{5}.$$

$$\tan \theta = \frac{3/5}{-4/5} = -\frac{3}{4}.$$

$$\cos = -\frac{4}{5}, \tan = -\frac{3}{4}$$

Exercise 13 - Find sine in quadrant III

$$\sin^2 \theta = 1 - \frac{25}{169} = \frac{144}{169}. \text{ Sine is negative in quadrant III: } \sin \theta = -\frac{12}{13}.$$

$$\tan \theta = \frac{-12/13}{-5/13} = \frac{12}{5}.$$

$$\sin = -\frac{12}{13}, \tan = \frac{12}{5}$$

Exercise 14 - Quadrant IV

$$\cos^2 \theta = 1 - \frac{1}{4} = \frac{3}{4}. \text{ Cosine is positive in quadrant IV: } \frac{\sqrt{3}}{2}.$$

$$\tan \theta = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

$$\cos = \frac{\sqrt{3}}{2}, \tan = -\frac{\sqrt{3}}{3}$$

Exercise 15 - Simplify with the identity

$$\sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x). \text{ Cancel } 1 - \cos x.$$

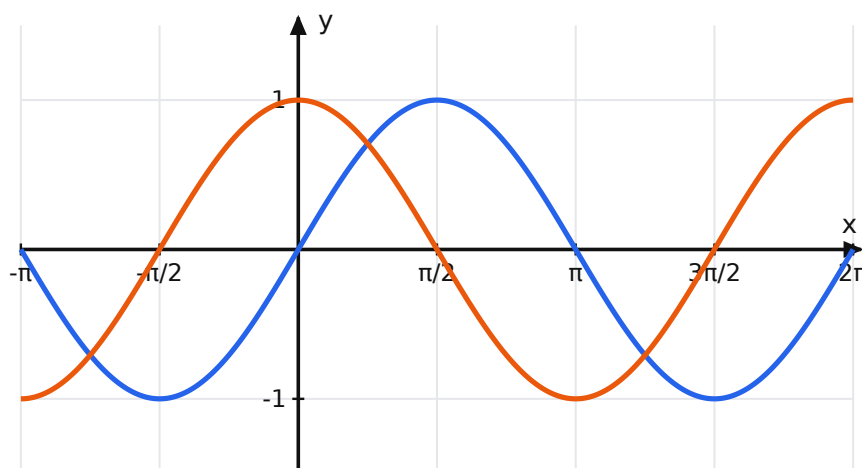
$$1 + \cos x$$

Exercise 16 - Prove an identity

$(1 - \sin x)(1 + \sin x) = 1 - \sin^2 x$. By the identity, $1 - \sin^2 x = \cos^2 x$.

Both sides equal $\cos^2 x$

Exercise 17 - Cosine is shifted sine



$y = \sin x$ (blue) and $y = \cos x$ (orange) both have period 2π and amplitude 1.

Shifting the sine graph left $\frac{\pi}{2}$ moves its maximum from $x = \frac{\pi}{2}$ to $x = 0$, which is where cosine has its maximum 1.

The two graphs coincide after a left shift of $\pi/2$

Exercise 18 - Daylight

Amplitude 2.5, midline 12: maximum 14.5 and minimum 9.5 hours.

Period $\frac{2\pi}{2\pi/365} = 365$ days.

14.5 h, 9.5 h; period 365 days

Exercise 19 - Find the error

The period is $\frac{2\pi}{B}$, not $B \cdot \pi$. With $B = 3$: $\frac{2\pi}{3}$. The graph completes three cycles in 2π .

$$\frac{2\pi}{3}$$

Exercise 20 - Spring

Amplitude $\frac{18-6}{2}=6$, midline $\frac{18+6}{2}=12$, $B=\frac{2\pi}{3}$.

Starting at the minimum means a reflected cosine: $y(0)=-6+12=6$.

$$y(t)=-6\cos\left(\frac{2\pi}{3}t\right)+12$$