

Algebra 2 - Radians and the Unit Circle - Lesson

Common Core F-TF.A.1, F-TF.A.2, F-TF.A.3 · maths-pdf.fr/en/

A radian is the central angle whose arc length equals the radius, so a full turn is 2π radians and 180 degrees equals π radians. On the unit circle, the point at angle θ is $(\cos \theta, \sin \theta)$, which defines sine and cosine for every real number. Reference angles give exact values.

Algebra 2 · Common Core State Standards: **F-TF.A.1, F-TF.A.2, F-TF.A.3**

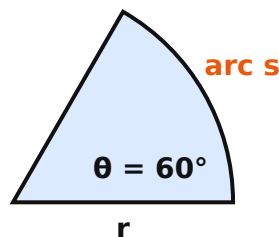
Key facts

- $\theta = s / r$, and $180^\circ = \pi$ radians.
- Arc length $s = r\theta$ when θ is in radians.
- Point on the unit circle at θ : $(\cos \theta, \sin \theta)$.
- $\tan \theta = \sin \theta / \cos \theta$; $\cos^2 \theta + \sin^2 \theta = 1$.
- Use reference angles and quadrant signs for exact values.

What is a radian?

Radian measure

One **radian** is the measure of a central angle whose arc has the same length as the radius. In general, $\theta = \frac{s}{r}$, where s is the arc length and r the radius. A full turn is 2π radians, so $180^\circ = \pi$ radians.

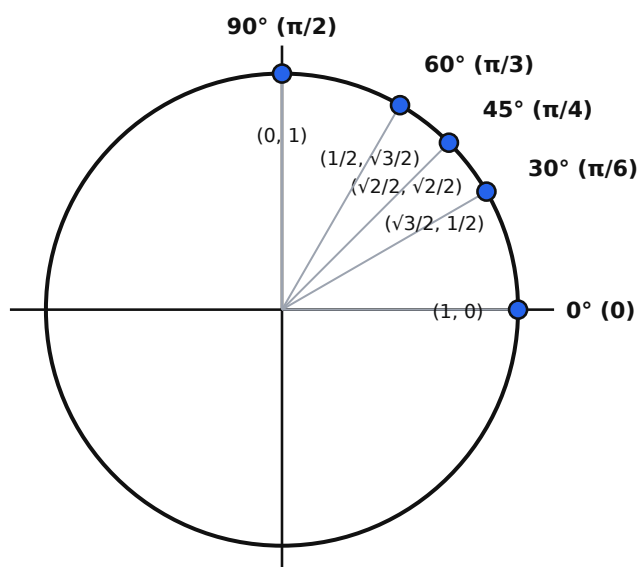


A sector: $s = r$ times the central angle in radians.

To convert, multiply degrees by $\frac{\pi}{180}$ to get radians, or radians by $\frac{180}{\pi}$ to get degrees. For example $60^\circ = \frac{\pi}{3}$, $225^\circ = \frac{5\pi}{4}$, and $\frac{7\pi}{6} = 210^\circ$. The arc length formula becomes $s = r\theta$: a circle of radius 6 inches and angle $\frac{\pi}{3}$ has arc length $6 \cdot \frac{\pi}{3} = 2\pi$ inches.

How is the unit circle used to define sine and cosine?

The **unit circle** has radius 1 and center at the origin. For an angle θ measured counterclockwise from the positive x-axis, the terminal point is $(\cos\theta, \sin\theta)$, and $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Because the point lies on the circle, $\cos^2\theta + \sin^2\theta = 1$.



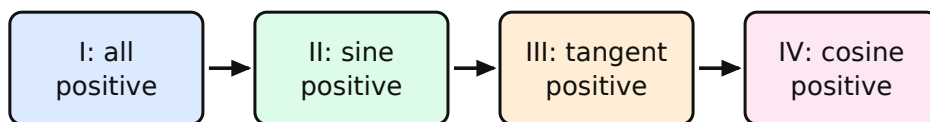
Quadrant I: the points $(\cos \theta, \sin \theta)$ for the special angles.

Memorize the first quadrant. The point for $\frac{\pi}{6}$ is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$, for $\frac{\pi}{4}$ it is $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, and for $\frac{\pi}{3}$ it is $(\frac{1}{2}, \frac{\sqrt{3}}{2})$.

How do you find trig values in other quadrants?

Use the **reference angle**: the acute angle between the terminal side and the x-axis. Find the value for the reference angle, then choose the sign from the quadrant. This works for all real numbers, because angles larger than 2π or negative angles wrap around the circle:

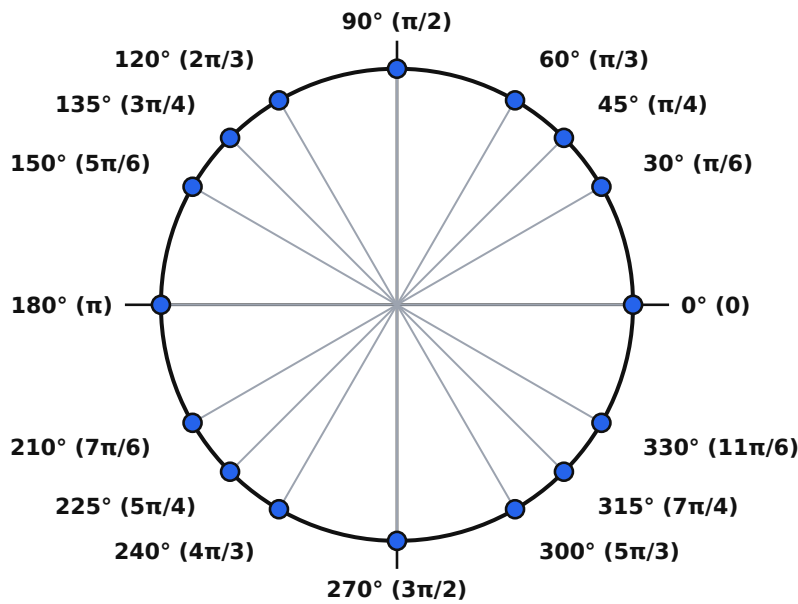
$$\sin \frac{13\pi}{6} = \sin \frac{\pi}{6} = \frac{1}{2}.$$



Signs by quadrant: All, Sine, Tangent, Cosine.

1. Reduce the angle to between 0 and 2π by adding or subtracting full turns.
2. Find the reference angle.
3. Evaluate the function at the reference angle.
4. Apply the sign for the quadrant.

Example: $\cos \frac{5\pi}{6}$: the angle is in quadrant II, the reference angle is $\frac{\pi}{6}$, $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, and cosine is negative in quadrant II, so the answer is $-\frac{\sqrt{3}}{2}$.



Angles around the unit circle in degrees and radians.

Questions about radians and the unit circle

What is a radian?

A radian is the angle at the center of a circle that cuts off an arc equal in length to the radius. A full circle is 2π radians, which equals 360 degrees.

How do you convert degrees to radians?

Multiply the degrees by π over 180. For example 90 degrees times π over 180 equals π over 2.

What does the unit circle show?

The unit circle shows that the point at angle θ has coordinates $(\cos \theta, \sin \theta)$. It extends sine and cosine to every real-number angle.

What is a reference angle?

A reference angle is the acute angle between the terminal side of an angle and the x-axis. Trig values at an angle equal the values at its reference angle, up to a sign.

What is the arc length formula?

Arc length equals the radius times the central angle in radians: $s = r\theta$. The angle must be in radians.