

Algebra 2 - Radians and the Unit Circle - Answer Key

Common Core F-TF.A.1, F-TF.A.2, F-TF.A.3 · maths-pdf.fr/en/

Every worksheet problem on radians and the unit circle is solved here, one step at a time. Compare your method as well as your final answer, because the steps show where using the wrong sign in a quadrant can go wrong. Final answers appear in bold so you can find them quickly when you only need a check.

Algebra 2 · Common Core State Standards: **F-TF.A.1, F-TF.A.2, F-TF.A.3**

Exercise 1 - Degrees to radians

Multiply by $\frac{\pi}{180}$: $60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$.

$$\frac{\pi}{3}$$

Exercise 2 - A larger angle

$225 \cdot \frac{\pi}{180} = \frac{5\pi}{4}$, since $\frac{225}{180} = \frac{5}{4}$.

$$\frac{5\pi}{4}$$

Exercise 3 - Radians to degrees

Multiply by $\frac{180}{\pi}$: $\frac{7\pi}{6} \cdot \frac{180}{\pi} = 210$.

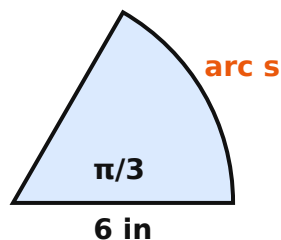
$$210^\circ$$

Exercise 4 - Decimal radians

$2 \cdot \frac{180}{\pi} \approx 114.6$.

$$114.6^\circ$$

Exercise 5 - Arc length



A sector: $s = r$ times the central angle in radians.

$$s = r\theta = 6 \cdot \frac{\pi}{3} = 2\pi.$$

$$2\pi \approx 6.28.$$

2π inches, about 6.28 in

Exercise 6 - Find the angle

$$\theta = \frac{s}{r} = \frac{15}{10} = 1.5.$$

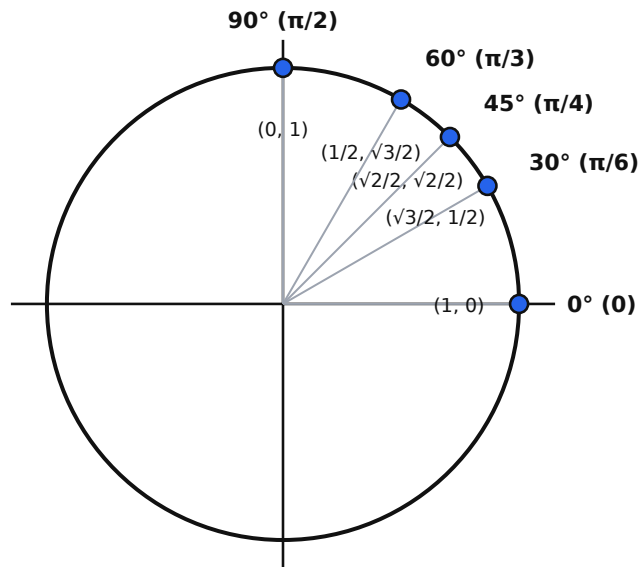
1.5 radians

Exercise 7 - Axis angle

The terminal side points straight up: (0,1).

(0,1); **cos = 0, sin = 1**

Exercise 8 - Angle of 30 degrees



Quadrant I: the points $(\cos \theta, \sin \theta)$ for the special angles.

The point is $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. $\tan \frac{\pi}{6} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

$$\sin = \frac{1}{2}, \cos = \frac{\sqrt{3}}{2}, \tan = \frac{\sqrt{3}}{3}$$

Exercise 9 - Angle of 45 degrees

The point is $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, so the tangent is the ratio 1.

$$\sin = \frac{\sqrt{2}}{2}, \cos = \frac{\sqrt{2}}{2}, \tan = 1$$

Exercise 10 - Quadrant II sine

$\frac{2\pi}{3} = 120^\circ$ is in quadrant II with reference angle $\frac{\pi}{3}$. Sine is positive in quadrant II.

$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

Exercise 11 - Quadrant II cosine

Reference angle $\frac{\pi}{6}$; cosine is negative in quadrant II.

$$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

Exercise 12 - Quadrant IV sine

$\frac{7\pi}{4} = 315^\circ$, quadrant IV, reference angle $\frac{\pi}{4}$. Sine is negative there.

$$\sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$$

Exercise 13 - Tangent in quadrant II

$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$. Divide.

$$\tan \frac{3\pi}{4} = -1$$

Exercise 14 - Straight angle

The point at π is $(-1,0)$. Tangent is $\frac{0}{-1} = 0$.

$$\sin \pi = 0, \cos \pi = -1, \tan \pi = 0$$

Exercise 15 - Beyond one turn

$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}$. The point is the same as at $\frac{\pi}{6}$.

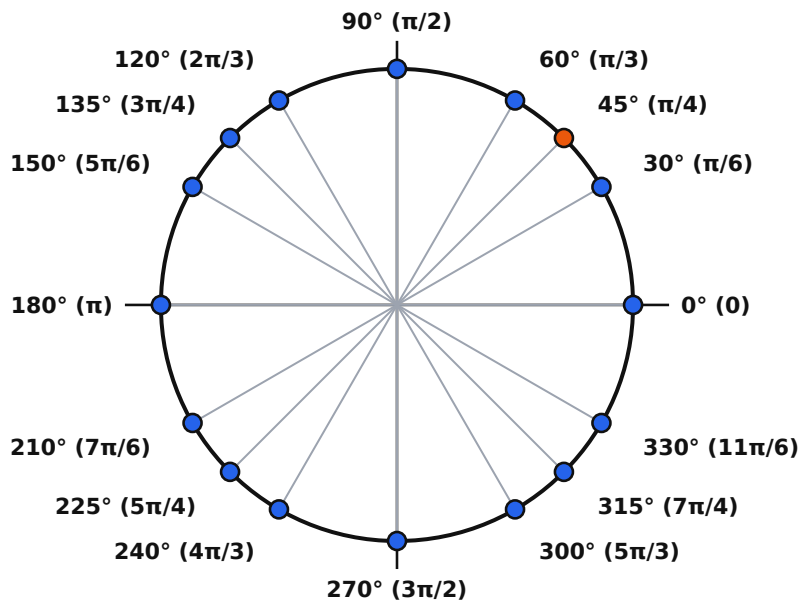
$$\sin \frac{13\pi}{6} = \frac{1}{2}$$

Exercise 16 - Negative angle

Negative angles turn clockwise. $-\frac{\pi}{3}$ ends in quadrant IV, so cosine is positive. $-\frac{\pi}{6}$ ends in quadrant IV, so sine is negative.

$$\frac{1}{2} \text{ and } -\frac{1}{2}$$

Exercise 17 - Solve $\sin \theta$



Angles around the unit circle in degrees and radians.

The reference angle is $\frac{\pi}{4}$. Sine is positive in quadrants I and II.

$$\theta = \frac{\pi}{4} \text{ or } \pi - \frac{\pi}{4} = \frac{3\pi}{4}.$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

Exercise 18 - Solve $\cos \theta$

The reference angle is $\frac{\pi}{3}$. Cosine is negative in quadrants II and III.

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \text{ or } \pi + \frac{\pi}{3} = \frac{4\pi}{3}.$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

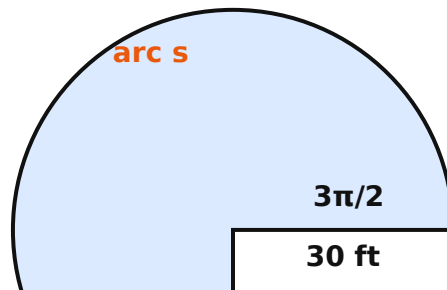
Exercise 19 - Identify the angle

The cosine is negative and the sine positive: quadrant II. The reference angle satisfies $\sin = \frac{1}{2}$: $\frac{\pi}{6}$.

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

$$\theta = \frac{5\pi}{6}$$

Exercise 20 - Ferris wheel



A sector: $s = r$ times the central angle in radians.

$$s = r\theta = 30 \cdot \frac{3\pi}{2} = 45\pi.$$

$$45\pi \approx 141.4 \text{ feet.}$$

$45\pi \text{ ft}$, about 141.4 ft

Exercise 21 - Windshield wiper

$$120^\circ = \frac{2\pi}{3}. s = 18 \cdot \frac{2\pi}{3} = 12\pi.$$

$$12\pi \approx 37.7 \text{ inches.}$$

$12\pi \text{ in}$, about 37.7 in

Exercise 22 - Find the error

$\frac{5\pi}{6}$ is in quadrant II, where sine is positive. The reference angle is $\frac{\pi}{6}$.

$$\sin \frac{5\pi}{6} = \frac{1}{2}$$