

AP Calculus AB - Average Value, Area and Volume - Answer Key

Common Core AP CALC AB 8.1, AP CALC AB 8.2, AP CALC AB 8.3, AP CALC AB 8.4, AP CALC AB 8.5, AP CALC AB 8.6, AP CALC AB 8.7, AP CALC AB 8.8, AP CALC AB 8.9, AP CALC AB 8.10, AP CALC AB 8.11, AP CALC AB 8.12 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the area and volume worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

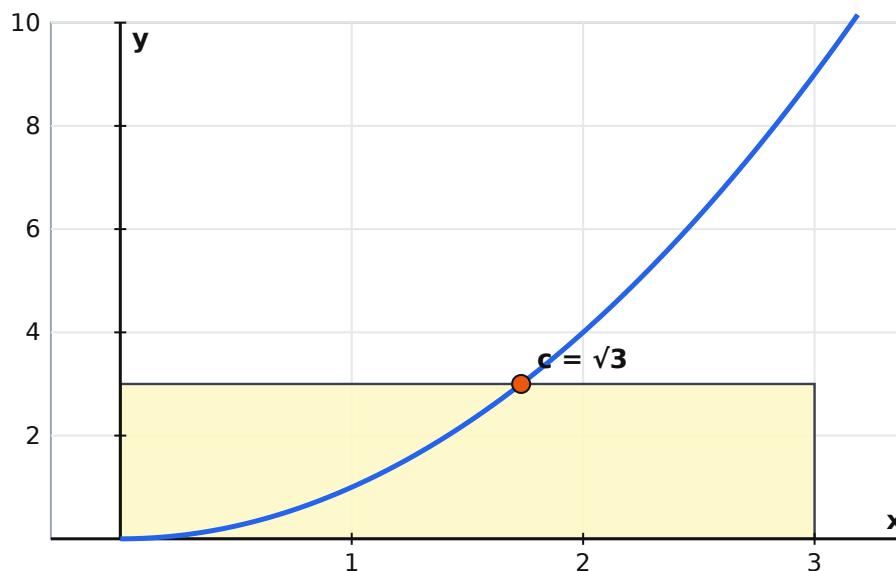
AP Calculus AB · Common Core State Standards: **AP CALC AB 8.1, AP CALC AB 8.2, AP CALC AB 8.3, AP CALC AB 8.4, AP CALC AB 8.5, AP CALC AB 8.6, AP CALC AB 8.7, AP CALC AB 8.8, AP CALC AB 8.9, AP CALC AB 8.10, AP CALC AB 8.11, AP CALC AB 8.12**

Exercise 1 - Average value of x squared

$$\frac{1}{3} \int_0^3 x^2 dx = \frac{1}{3} \cdot \frac{27}{3} = 3.$$

Solve $c^2 = 3$: $c = \sqrt{3} \approx 1.732$ (in $[0, 3]$).

Final answer: $f_{\text{avg}} = 3$, $c = \sqrt{3}$.



The curve crosses the average height at $x = \sqrt{3}$.

Exercise 2 - Average value of a root

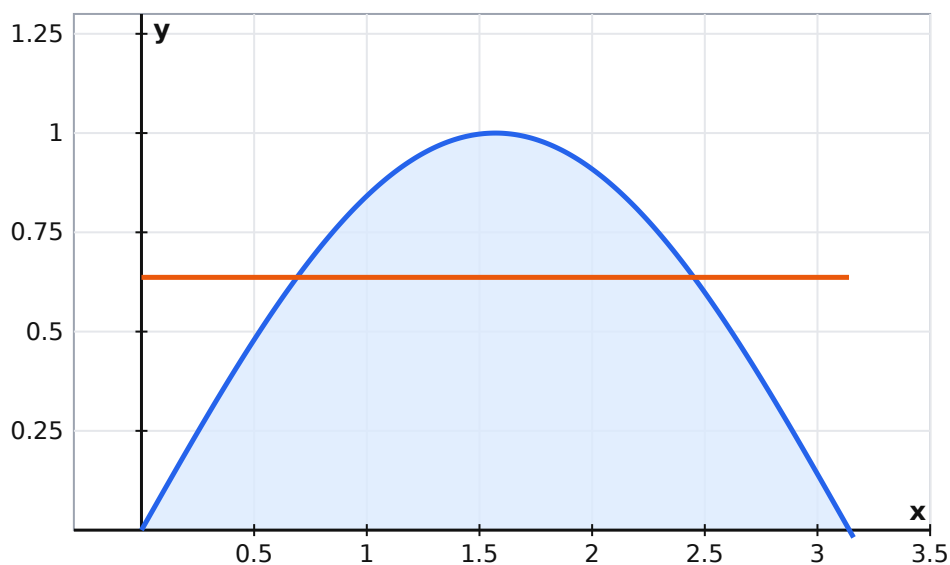
$$\frac{1}{4} \int_0^4 x^{1/2} dx = \frac{1}{4} \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{1}{4} \cdot \frac{16}{3} = \frac{4}{3}.$$

Final answer: $\frac{4}{3}$.

Exercise 3 - Average value of sine

$$\frac{1}{\pi} \int_0^{\pi} \sin x \, dx = \frac{1}{\pi} [-\cos x]_0^{\pi} = \frac{2}{\pi}.$$

Final answer: $\frac{2}{\pi} \approx 0.637$.



The average value of $\sin x$ on $[0, \pi]$ is $2/\pi$.

Exercise 4 - Average temperature

$$\frac{1}{24} \int_0^{24} \left(60 + 10 \sin \frac{\pi t}{12} \right) dt = \frac{1}{24} \left[60t - \frac{120}{\pi} \cos \frac{\pi t}{12} \right]_0^{24}.$$

The cosine terms cancel because $\cos 2\pi = \cos 0 = 1$, leaving $\frac{1}{24}(1440) = 60$.

Final answer: 60°F .

Exercise 5 - Average value from a table

$$\text{Integral estimate: } 2 \left[\frac{3+5}{2} + \frac{5+9}{2} + \frac{9+11}{2} + \frac{11+10}{2} \right] = 2(4+7+10+10.5) = 63.$$

Average: $\frac{63}{8} = 7.875$.

Final answer: ≈ 7.9 m/s.

Exercise 6 - Displacement and distance

$v = (t-1)(t-3)$ changes sign at $t=1$ and $t=3$.

Displacement: $\int_0^4 v \, dt = \left[\frac{t^3}{3} - 2t^2 + 3t \right]_0^4 = \frac{64}{3} - 32 + 12 = \frac{4}{3}$.

Distance: $\int_0^1 = \frac{4}{3}$, $\left| \int_1^3 \right| = \frac{4}{3}$, $\int_3^4 = \frac{4}{3}$. Sum 4.

Final answer: displacement $\frac{4}{3}$ ft; distance 4 ft.

Exercise 7 - Position from velocity

$s(3) = s(0) + \int_0^3 (2t+1) \, dt = 5 + [t^2 + t]_0^3 = 5 + 12 = 17$.

Final answer: 17 m.

Exercise 8 - Velocity from acceleration

$v(3) = 2 + \int_0^3 6t \, dt = 2 + [3t^2]_0^3 = 2 + 27 = 29$.

Final answer: 29 m/s.

Exercise 9 - Net change of a population

Net change: $\int_0^{10} (40-6t) \, dt = [40t - 3t^2]_0^{10} = 400 - 300 = 100$. Population: $300 + 100 = 400$.

$r = 0$ at $t = \frac{20}{3}$, changing from positive to negative, so the maximum is at $t \approx 6.67$ weeks.

Final answer: 400 fish; maximum at $t = \frac{20}{3}$.

Exercise 10 - Area between a root and a parabola

They meet where $\sqrt{x} = x^2$: $x=0$ and $x=1$. The root curve is on top.

$$\int_0^1 (\sqrt{x} - x^2) dx = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}.$$

Final answer: $\frac{1}{3}$.

Exercise 11 - Area between two parabolas

$$8 - x^2 = x^2 \text{ gives } x = \pm 2.$$

$$\int_{-2}^2 (8 - 2x^2) dx = \left[8x - \frac{2x^3}{3} \right]_{-2}^2 = 2 \left(16 - \frac{16}{3} \right) = \frac{64}{3}.$$

Final answer: $\frac{64}{3}$.

Exercise 12 - Area under a line above an exponential

They meet where $e^x = 2$: $x = \ln 2$. The line is on top for $0 \leq x \leq \ln 2$.

$$\int_0^{\ln 2} (2 - e^x) dx = [2x - e^x]_0^{\ln 2} = (2 \ln 2 - 2) - (0 - 1) = 2 \ln 2 - 1.$$

Final answer: $2 \ln 2 - 1 \approx 0.386$.

Exercise 13 - Sine and cosine

On this interval $\cos x \geq \sin x$.

$$[\sin x + \cos x]_0^{\pi/4} = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - 1 = \sqrt{2} - 1.$$

Final answer: $\sqrt{2} - 1 \approx 0.414$.

Exercise 14 - Integrating with respect to y

Intersections: $y^2 = y + 2$ gives $y = -1$ and $y = 2$. The line is to the right.

$$\int_{-1}^2 (y + 2 - y^2) dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^2 = \frac{10}{3} - \left(-\frac{7}{6} \right) = \frac{9}{2}.$$

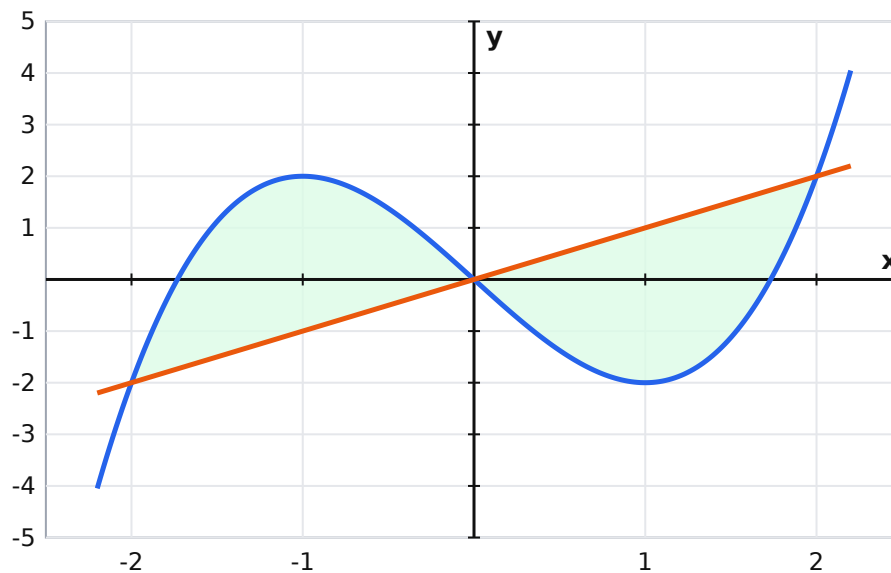
Final answer: $\frac{9}{2}$.

Exercise 15 - Two enclosed regions

$x^3 - 3x = x$ gives $x(x^2 - 4) = 0$, so $x = -2, 0, 2$. For $0 < x < 2$ the line is on top.

By symmetry: $2 \int_0^2 (x - (x^3 - 3x)) dx = 2 \int_0^2 (4x - x^3) dx = 2 \left[2x^2 - \frac{x^4}{4} \right]_0^2 = 2(8 - 4) = 8$.

Final answer: 8.



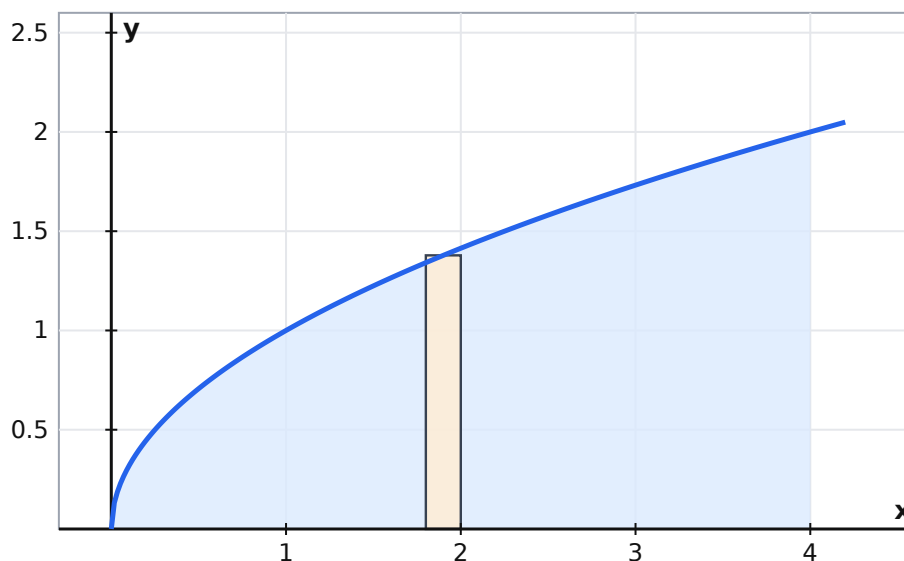
Two regions of area 4 each.

Exercise 16 - Square cross sections

A square has side $\sqrt{x}$, so $A(x) = x$.

$$V = \int_0^4 x \, dx = 8.$$

Final answer: 8.



The base and one slice.

Exercise 17 - Rectangles on an exponential base

Each rectangle has width e^x and height 3, so $A(x) = 3e^x$.

$$V = \int_0^1 3e^x dx = 3[e^x]_0^1 = 3(e-1).$$

Final answer: $3(e-1) \approx 5.15$.

Exercise 18 - Disc method about the x-axis

Each disc has radius $\sqrt{x}$: $V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \int_0^4 x dx = \pi \cdot 8$.

Final answer: $8\pi \approx 25.13$.

Exercise 19 - Washer with a horizontal cap

Outer radius 3, inner radius x^2+1 .

$$V = \pi \int_0^{\sqrt{2}} (9 - (x^2+1)^2) dx = \pi \int_0^{\sqrt{2}} (8 - 2x^2 - x^4) dx = \pi \left[8x - \frac{2x^3}{3} - \frac{x^5}{5} \right]_0^{\sqrt{2}}.$$

$$= \pi \sqrt{2} \left(8 - \frac{4}{3} - \frac{4}{5} \right) = \frac{88\sqrt{2}}{15} \pi \approx 26.06.$$

Final answer: $\frac{88\sqrt{2}}{15} \pi \approx 26.06$.

Exercise 20 - Revolving about the y-axis

The region to the left of the curve is $x = \sqrt{y}$ for $0 \leq y \leq 4$. Discs of radius $\sqrt{y}$:

$$V = \pi \int_0^4 y \, dy = \pi \cdot 8 = 8\pi.$$

Final answer: $8\pi \approx 25.13$.

Exercise 21 - Volume of a cone

$$V = \pi \int_0^2 x^2 dx = \pi \cdot \frac{8}{3} = \frac{8\pi}{3}.$$

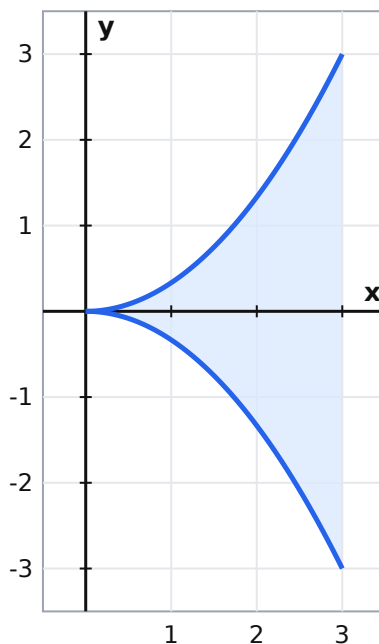
Cone with radius $r = 2$ and height $h = 2$: $\frac{1}{3}\pi(4)(2) = \frac{8\pi}{3}$. They agree.

Final answer: $\frac{8\pi}{3}$.

Exercise 22 - A vase

$$V = \pi \int_0^3 \left(\frac{x^2}{3}\right)^2 dx = \frac{\pi}{9} \int_0^3 x^4 dx = \frac{\pi}{9} \cdot \frac{243}{5} = \frac{27\pi}{5}.$$

Final answer: $\frac{27\pi}{5} \approx 16.96 \text{ in}^3$.



The vase: the curve and its mirror image.

Exercise 23 - Average rate of production

$$\int_0^6 (50 + 6t - t^2) dt = \left[50t + 3t^2 - \frac{t^3}{3} \right]_0^6 = 300 + 108 - 72 = 336.$$

Average rate: $\frac{336}{6} = 56$ items per hour.

Final answer: 336 items; 56 items/hour.