

AP Calculus AB - Average Value, Area and Volume - Worksheet

Common Core AP CALC AB 8.1, AP CALC AB 8.2, AP CALC AB 8.3, AP CALC AB 8.4, AP CALC AB 8.5, AP CALC AB 8.6, AP CALC AB 8.7, AP CALC AB 8.8, AP CALC AB 8.9, AP CALC AB 8.10, AP CALC AB 8.11, AP CALC AB 8.12 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 23 problems practice average value, motion with integrals, area between curves, volumes with cross sections and discs and washers (AP Calculus AB, topics 8.1 to 8.12). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems cover curves that cross, integration in y, cross sections and washers, and real situations such as a vase and factory output. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 8.1, AP CALC AB 8.2, AP CALC AB 8.3, AP CALC AB 8.4, AP CALC AB 8.5, AP CALC AB 8.6, AP CALC AB 8.7, AP CALC AB 8.8, AP CALC AB 8.9, AP CALC AB 8.10, AP CALC AB 8.11, AP CALC AB 8.12**

Exercise 1 - Average value of x squared

Find the average value of $f(x) = x^2$ on $[0, 3]$ and the value c where $f(c)$ equals it.

Exercise 2 - Average value of a root

Find the average value of $f(x) = \sqrt{x}$ on $[0, 4]$.

Exercise 3 - Average value of sine

Find the average value of $\sin x$ on $[0, \pi]$.

Exercise 4 - Average temperature

The temperature in a greenhouse in °F is $T(t) = 60 + 10\sin\frac{\pi t}{12}$ for $0 \leq t \leq 24$ hours. Find the average temperature.

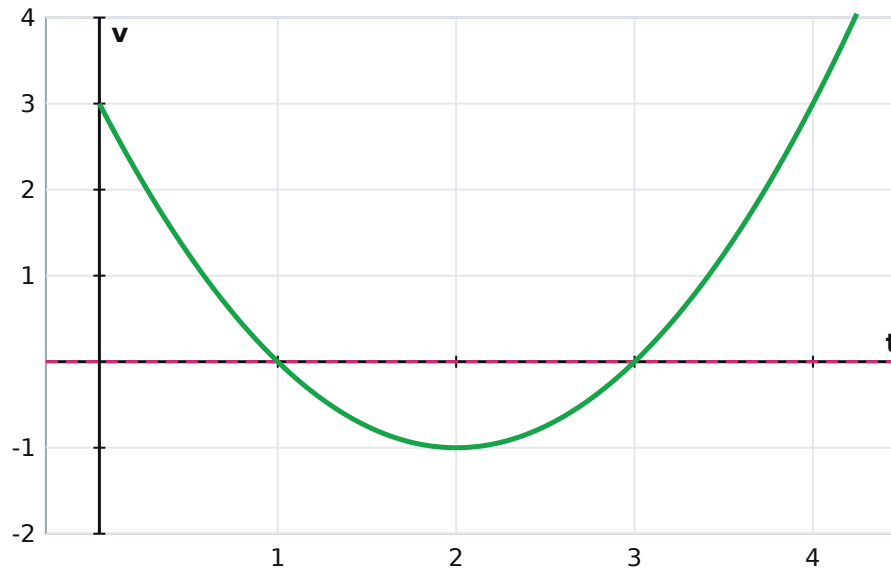
Exercise 5 - Average value from a table

t (s)	0	2	4	6	8
v (m/s)	3	5	9	11	10

Velocity data.

Use a trapezoidal sum with four subintervals to estimate the average velocity on $[0,8]$.

Exercise 6 - Displacement and distance



Velocity of a skater in feet per second.

A skater has velocity $v(t) = t^2 - 4t + 3$ ft/s for $0 \leq t \leq 4$. Find the displacement and the total distance.

Exercise 7 - Position from velocity

A robot has velocity $v(t) = 2t + 1$ m/s and position $s(0) = 5$ m. Find $s(3)$.

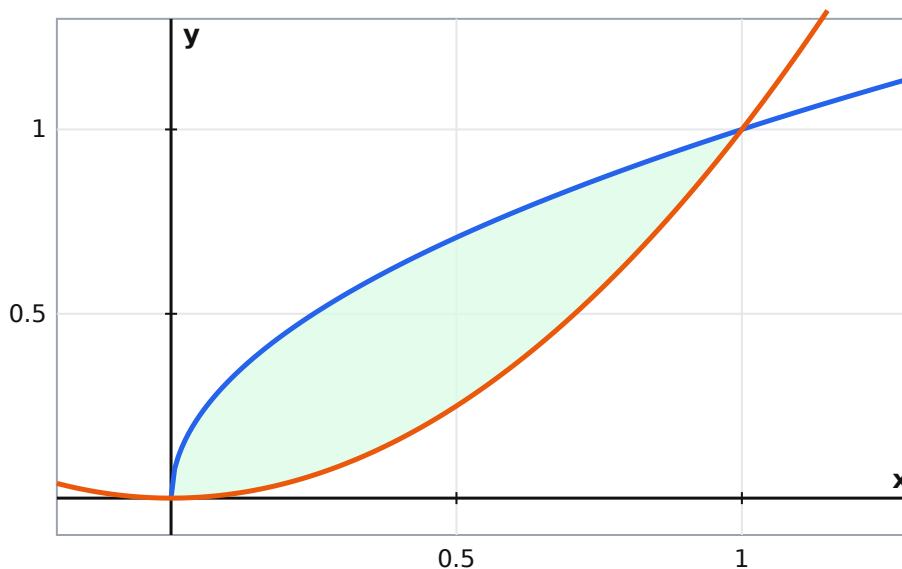
Exercise 8 - Velocity from acceleration

A car has acceleration $a(t) = 6t$ m/s² and $v(0) = 2$ m/s. Find $v(3)$.

Exercise 9 - Net change of a population

A fish population changes at the rate $r(t) = 40 - 6t$ fish per week for $0 \leq t \leq 10$. The population at $t = 0$ is 300. Find the population at $t = 10$ and the week of the maximum population.

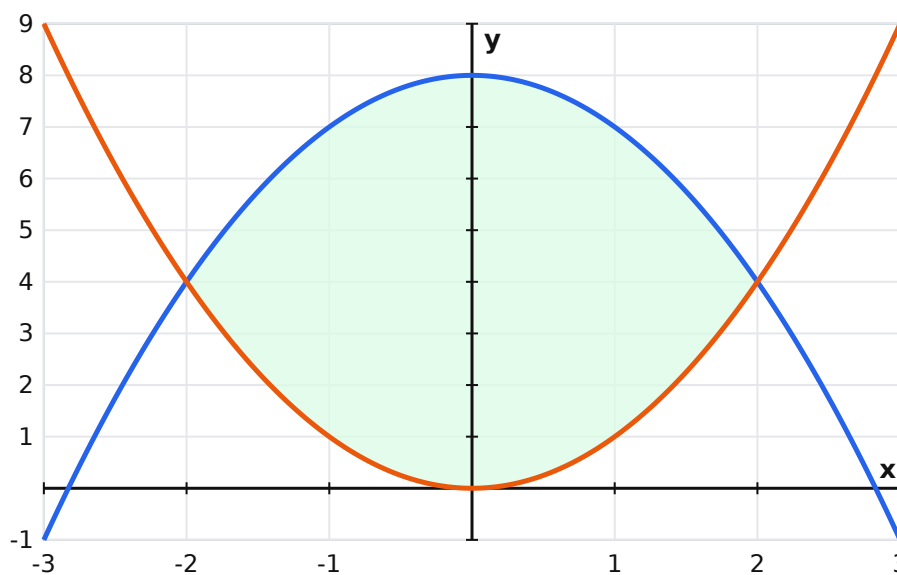
Exercise 10 - Area between a root and a parabola



The region between $y = \sqrt{x}$ and $y = x^2$.

Find the area of the region between $y = \sqrt{x}$ and $y = x^2$.

Exercise 11 - Area between two parabolas



Two parabolas intersecting at $x = \pm 2$.

Find the area between $y = 8 - x^2$ and $y = x^2$.

Exercise 12 - Area under a line above an exponential

Find the area bounded by $y=2$, $y=e^x$ and the y -axis.

Exercise 13 - Sine and cosine

Find the area between $y=\cos x$ and $y=\sin x$ from $x=0$ to $x=\frac{\pi}{4}$.

Exercise 14 - Integrating with respect to y

Find the area between $x=y^2$ and $x=y+2$.

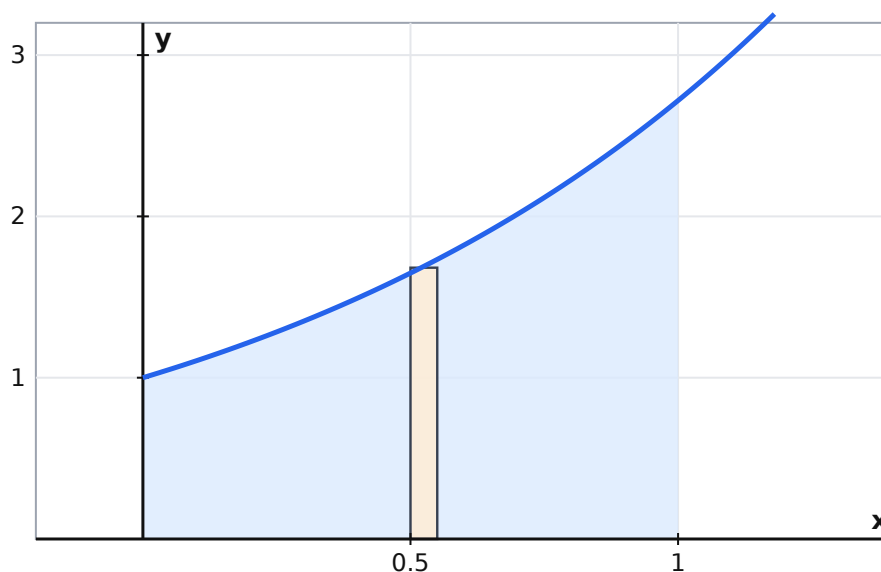
Exercise 15 - Two enclosed regions

Find the total area enclosed by $y=x^3-3x$ and $y=x$.

Exercise 16 - Square cross sections

The base of a solid is the region under $y=\sqrt{x}$ on $[0,4]$. Cross sections perpendicular to the x -axis are squares. Find the volume.

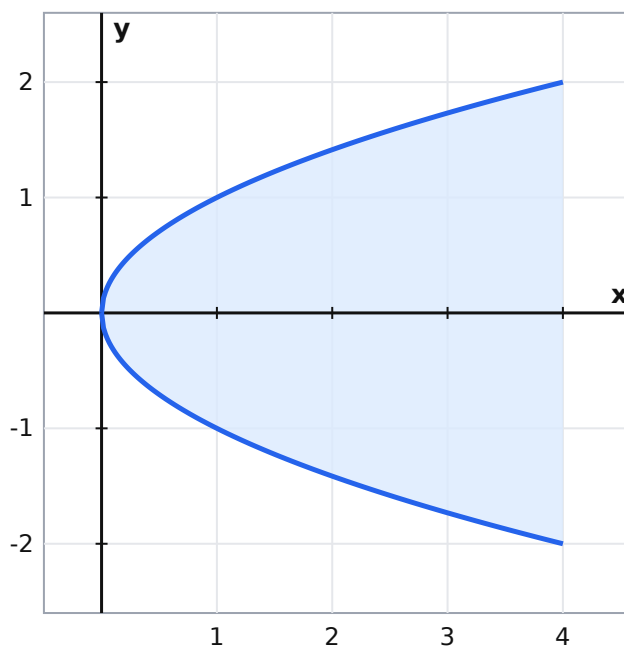
Exercise 17 - Rectangles on an exponential base



The base region under e^x .

The base of a solid is the region under $y=e^x$ on $[0,1]$. Cross sections perpendicular to the x -axis are rectangles of height 3. Find the volume.

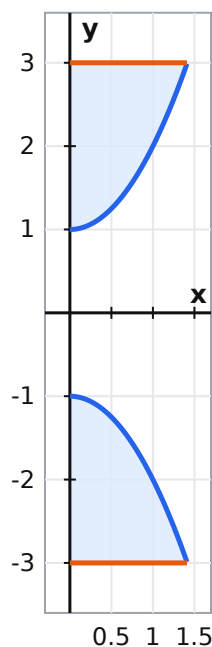
Exercise 18 - Disc method about the x-axis



The solid of revolution.

Find the volume of the solid formed by revolving the region under $y = \sqrt{x}$, $0 \leq x \leq 4$, about the x -axis.

Exercise 19 - Washer with a horizontal cap



Outer radius 3, inner radius $x^2 + 1$.

The region between $y = x^2 + 1$ and $y = 3$ for $0 \leq x \leq \sqrt{2}$ is revolved about the x -axis. Find the volume.

Exercise 20 - Revolving about the y -axis

The region under $y = x^2$ for $0 \leq x \leq 2$ is revolved about the y -axis. Use discs in y to find the volume of the solid inside the curve up to $y = 4$ (the region between the y -axis and the curve).

Exercise 21 - Volume of a cone

Revolve $y = x$ for $0 \leq x \leq 2$ about the x -axis and verify the cone volume formula $V = \frac{1}{3}\pi r^2 h$.

Exercise 22 - A vase

A vase is made by revolving the curve $y = \frac{x^2}{3}$ (inches), $0 \leq x \leq 3$, about the x -axis. Find the volume of the solid it encloses.

Exercise 23 - Average rate of production

A factory's output rate is $r(t) = 50 + 6t - t^2$ items per hour for $0 \leq t \leq 6$ hours. Find the average rate and the total number of items.