

AP Calculus AB - Average Value, Area and Volume - Test Answer Key

Common Core AP CALC AB 8.1, AP CALC AB 8.2, AP CALC AB 8.3, AP CALC AB 8.4, AP CALC AB 8.5, AP CALC AB 8.6, AP CALC AB 8.7, AP CALC AB 8.8, AP CALC AB 8.9, AP CALC AB 8.10, AP CALC AB 8.11, AP CALC AB 8.12 · maths-pdf.fr/en/

Use this answer key to score the area and volume test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 8.1, AP CALC AB 8.2, AP CALC AB 8.3, AP CALC AB 8.4, AP CALC AB 8.5, AP CALC AB 8.6, AP CALC AB 8.7, AP CALC AB 8.8, AP CALC AB 8.9, AP CALC AB 8.10, AP CALC AB 8.11, AP CALC AB 8.12**

Question 1 - Average value [2 pts]

$$\int_0^{\ln 3} e^{2x} dx = \left[\frac{e^{2x}}{2} \right]_0^{\ln 3} = \frac{9-1}{2} = 4. \text{ Divide by the length } \ln 3.$$

$$\text{Final answer: } \frac{4}{\ln 3} \approx 3.64.$$

Scoring: 1 point for the integral; 1 point for dividing by the length.

Question 2 - Motion [2 pts]

$$\text{Displacement: } \int_0^5 (6-2t) dt = [6t-t^2]_0^5 = 5.$$

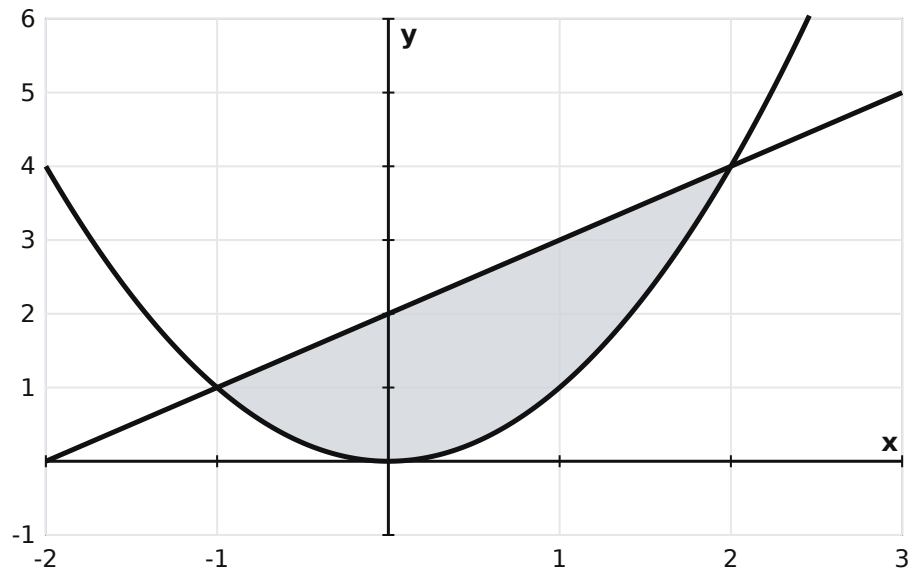
$$v=0 \text{ at } t=3. \text{ Distance: } \int_0^3 = 9 \text{ and } \left| \int_3^5 \right| = |(30-25)-9| = 4, \text{ total } 13.$$

Scoring: 1 point for the displacement; 1 point for the distance.

Question 3 - Area between curves [3 pts]

$x+2=x^2$ gives $x=-1$ and $x=2$. The line is on top.

$$\int_{-1}^2 (x+2-x^2) dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 = \frac{10}{3} - \left(-\frac{7}{6} \right) = \frac{9}{2}.$$



The region between $y = x + 2$ and $y = x^2$.

Scoring: 1 point for the intersections; 1 point for the integral; 1 point for the value.

Question 4 - Area with a crossing [3 pts]

They cross where $\sin x = \cos x$, at $x = \frac{\pi}{4}$. Cosine is on top before it, sine after it.

$$\int_0^{\pi/4} (\cos x - \sin x) dx = \sqrt{2} - 1 \text{ and } \int_{\pi/4}^{\pi} (\sin x - \cos x) dx = [-\cos x - \sin x]_{\pi/4}^{\pi} = 1 + \sqrt{2}.$$

Final answer: $2\sqrt{2} \approx 2.83$.

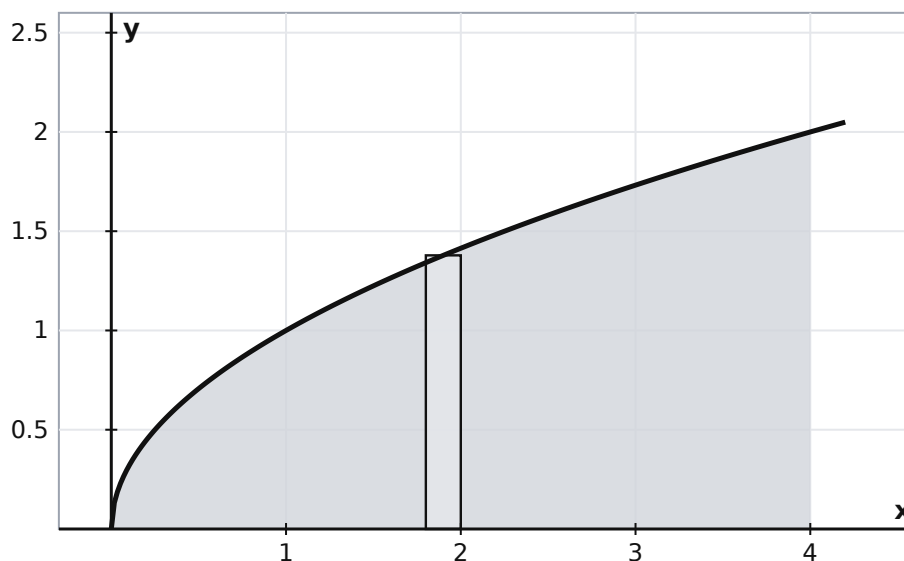
Scoring: 1 point for the crossing point; 1 point for the two integrals; 1 point for the total.

Question 5 - Cross sections [3 pts]

The side is $\sqrt{x}$, so the area is $\frac{\sqrt{3}}{4}s^2 = \frac{\sqrt{3}}{4}x$.

$$V = \frac{\sqrt{3}}{4} \int_0^4 x dx = \frac{\sqrt{3}}{4} \cdot 8 = 2\sqrt{3}.$$

Final answer: $2\sqrt{3} \approx 3.46$.



Base region under $y = \sqrt{x}$.

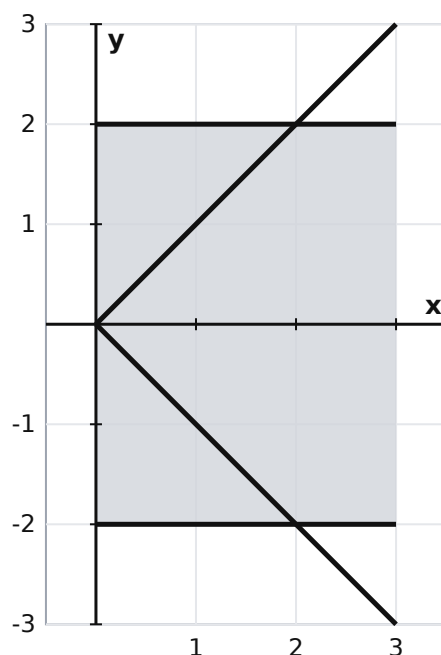
Scoring: 1 point for the area formula; 1 point for the integral; 1 point for the value.

Question 6 - Washer method [3 pts]

The curves meet at $x=2$. Outer radius 2, inner radius x for $0 \leq x \leq 2$.

$$V = \pi \int_0^2 (4 - x^2) dx = \pi \left[4x - \frac{x^3}{3} \right]_0^2 = \pi \left(8 - \frac{8}{3} \right) = \frac{16\pi}{3}.$$

Final answer: $\frac{16\pi}{3} \approx 16.76$.



Outer radius 2 and inner radius x.

Scoring: 1 point for radii; 1 point for the integral; 1 point for the value.

Question 7 - Average rate in context [2 pts]

$$\int_0^{30} (20 + 12 \sin \frac{\pi t}{30}) dt = \left[20t - \frac{360}{\pi} \cos \frac{\pi t}{30} \right]_0^{30} = 600 + \frac{720}{\pi} \approx 829.2.$$

Average: $\frac{829.2}{30} \approx 27.6$ cars per minute.

Scoring: 1 point for the total; 1 point for the average.

Question 8 - Volume of a solid of revolution [2 pts]

$$V = \pi \int_0^1 e^{2x} dx = \pi \left[\frac{e^{2x}}{2} \right]_0^1 = \frac{\pi}{2} (e^2 - 1).$$

Final answer: $\frac{\pi(e^2-1)}{2} \approx 10.04$.

Scoring: 1 point for the integral; 1 point for the value.

Total
20 points