

AP Calculus AB - Infinite Limits and Asymptotes - Lesson

Common Core AP CALC AB 1.13, AP CALC AB 1.14, AP CALC AB 1.15 · maths-pdf.fr/en/

An infinite limit means the outputs grow without bound near an input, which creates a vertical asymptote. A limit at infinity describes the value a function approaches as x grows without bound, which creates a horizontal asymptote. For rational functions, compare the degrees of the numerator and denominator to find the end behavior.

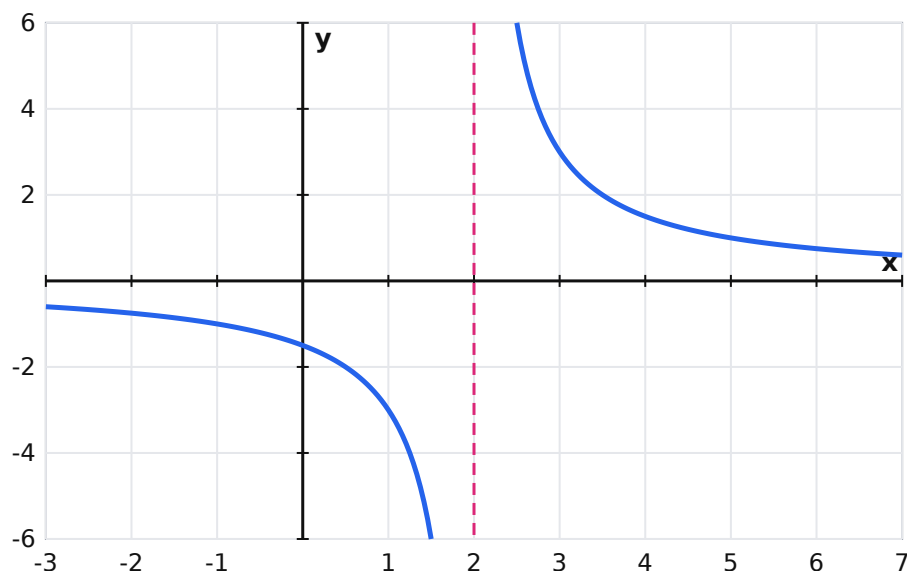
AP Calculus AB · Common Core State Standards: **AP CALC AB 1.13, AP CALC AB 1.14, AP CALC AB 1.15**

Key facts

- A vertical asymptote at $x = a$ occurs when a one-sided limit there is infinite.
- A horizontal asymptote $y = L$ comes from a limit at infinity equal to L .
- Rational functions: smaller top degree gives 0, equal degrees give the ratio of leading coefficients.
- Check the sign on each side of a vertical asymptote to decide between $+\infty$ and $-\infty$.

What is an infinite limit?

When outputs grow without bound near an input, we write $\lim_{x \rightarrow a} f(x) = \infty$ or $-\infty$. The limit does not exist as a number, but the symbol tells you the direction of the unbounded behavior. A function has a **vertical asymptote** at $x = a$ when at least one one-sided limit there is $\pm\infty$.

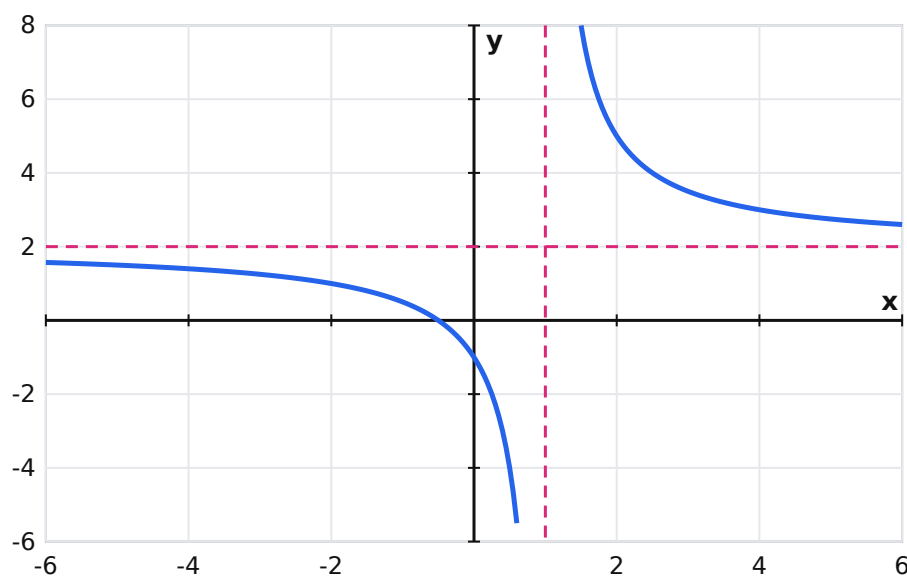


Near $x = 2$, the outputs go to $-\infty$ from the left and $+\infty$ from the right.

To find the sign, test the signs of the numerator and denominator just left and right of α . For $\frac{3}{x-2}$, the denominator is negative just left of 2 and positive just right of 2.

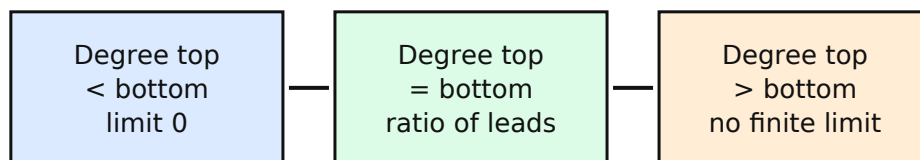
What are limits at infinity?

$\lim_{x \rightarrow \infty} f(x) = L$ means the outputs approach L as x grows without bound. Then the line $y = L$ is a **horizontal asymptote**. A function can have at most two horizontal asymptotes, one for each direction.



A vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 2$.

How do you find limits at infinity of rational functions?



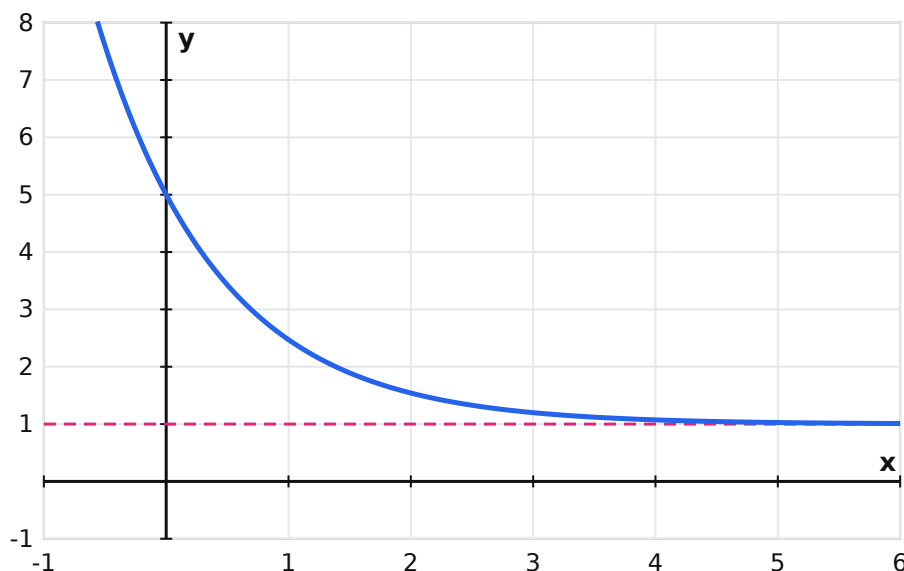
The three cases for a rational function as x approaches infinity.

Divide every term by the highest power of x in the denominator. For example,

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{5x^2 - 1} = \lim_{x \rightarrow \infty} \frac{3 + \frac{2}{x}}{5 - \frac{1}{x^2}} = \frac{3}{5}, \text{ because terms like } \frac{2}{x} \text{ approach } 0.$$

What about exponential and logarithmic functions?

Limit	Value
$\lim_{x \rightarrow \infty} e^x$	∞
$\lim_{x \rightarrow -\infty} e^x$	0
$\lim_{x \rightarrow \infty} e^{-x}$	0
$\lim_{x \rightarrow \infty} \ln x$	∞
$\lim_{x \rightarrow 0^+} \ln x$	$-\infty$



As x grows, $4e^{-x} + 1$ settles toward 1.

Example. The graph of $f(x) = 4e^{-x} + 1$ has horizontal asymptote $y = 1$ because $e^{-x} \rightarrow 0$ as $x \rightarrow \infty$.

How do you find every asymptote of a rational function?

1. Factor the numerator and the denominator completely, and cancel common factors. Each cancelled factor marks a hole, not an asymptote.
2. Set each remaining denominator factor equal to zero. Each solution is a vertical asymptote.
3. Compare the degrees of the numerator and denominator to find the horizontal asymptote, if there is one.
4. Check the sign on each side of every vertical asymptote to describe the graph.

For $\frac{x+2}{x^2-x-6}$, the factor $x+2$ cancels, so there is a hole at $x=-2$ and a vertical asymptote at $x=3$. The numerator has the smaller degree, so $y=0$ is the horizontal asymptote.

Questions about infinite limits and asymptotes

What does a limit of infinity mean?

It means the outputs increase without bound as x approaches the target. Strictly, the limit does not exist as a real number, but the notation describes the unbounded behavior.

How do you find a vertical asymptote?

Simplify the function, then find the x -values that make the denominator zero while the numerator stays nonzero. Factors that cancel leave a hole instead of an asymptote.

How do you find a horizontal asymptote of a rational function?

Compare degrees. If the numerator degree is smaller, $y = 0$. If the degrees are equal, y equals the ratio of the leading coefficients. If the numerator degree is larger, there is no horizontal asymptote.

Can a graph cross a horizontal asymptote?

Yes. A horizontal asymptote describes behavior as x grows very large, so the graph may cross it at moderate x -values before settling toward it.

What is the limit of $\sin(x)/x$ as x approaches infinity?

It is 0. Since $\sin x$ stays between -1 and 1 while x grows, the Squeeze Theorem traps $\sin(x)/x$ between $-1/x$ and $1/x$, and both bounds approach 0.