

AP Calculus AB - Infinite Limits and Asymptotes - Answer Key

Common Core AP CALC AB 1.13, AP CALC AB 1.14, AP CALC AB 1.15 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the asymptotes worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.13, AP CALC AB 1.14, AP CALC AB 1.15**

Exercise 1 - One-sided infinite limits

For $x > 3$ close to 3, $x - 3$ is a small positive number, so $\frac{1}{x-3} \rightarrow \infty$.

For $x < 3$, $x - 3$ is a small negative number, so the quotient goes to $-\infty$.

Final answer: ∞ from the right, $-\infty$ from the left, and **the two-sided limit does not exist**.

Exercise 2 - An even power

The square is positive on both sides of 2 and approaches 0, so the quotient is large and positive from both sides.

Final answer: ∞ (the outputs grow without bound, so there is a vertical asymptote at $x = 2$).

Exercise 3 - Sign analysis

Near $x = -1$, the numerator $x + 3$ is about 2 (positive).

From the left, $x + 1 < 0$ and small, so the quotient is negative and large: $-\infty$. From the right, $x + 1 > 0$, so the quotient goes to ∞ .

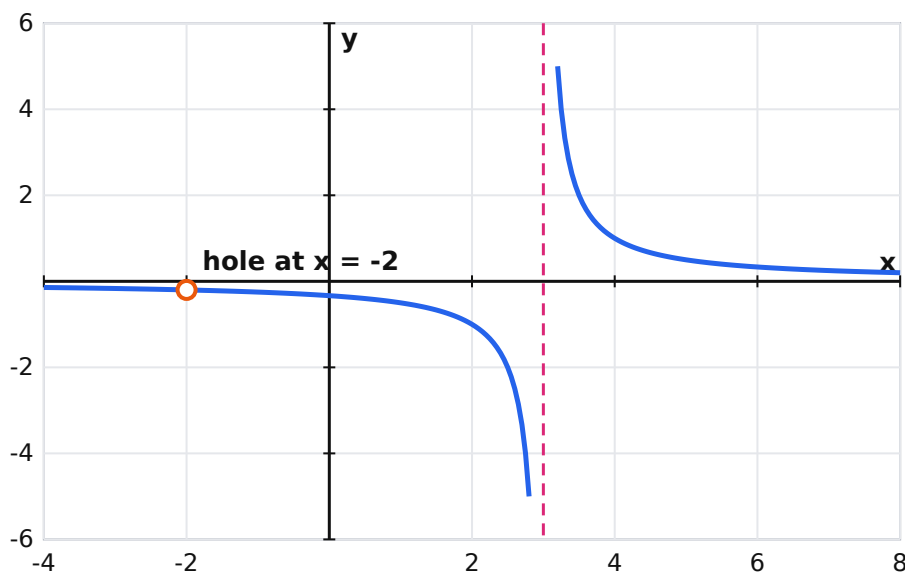
Final answer: $-\infty$ from the left and ∞ from the right.

Exercise 4 - Asymptote or hole?

Factor: $\frac{x+2}{(x-3)(x+2)}$. The factor $x+2$ cancels, leaving $\frac{1}{x-3}$.

At $x=3$ the denominator is zero and the numerator is 1, so there is a vertical asymptote. At $x=-2$ there is a hole at $(-2, -\frac{1}{5})$.

Final answer: **vertical asymptote $x = 3$ and a hole at $x = -2$.**



Simplified graph: asymptote at $x = 3$, hole at $x = -2$.

Exercise 5 - Asymptotes from a graph

Left of -1 the branch falls: $-\infty$. Right of -1 it rises: ∞ .

As $|x|$ grows, $\frac{2}{x+1} \rightarrow 0$ from both directions.

Final answer: $-\infty$, ∞ , 0, 0 (asymptotes $x = -1$ and $y = 0$).

Exercise 6 - Equal degrees

Divide top and bottom by x^2 : $\frac{3+\frac{2}{x}}{5-\frac{1}{x^2}}$. As $x \rightarrow \infty$, $\frac{2}{x} \rightarrow 0$ and $\frac{1}{x^2} \rightarrow 0$.

Final answer: $\frac{3}{5}$.

Exercise 7 - Difference of roots at infinity

Multiply by the conjugate: $\frac{(9x^2+1)-9x^2}{\sqrt{9x^2+1}+3x} = \frac{1}{\sqrt{9x^2+1}+3x}$.

The denominator grows without bound, so the quotient tends to 0.

Final answer: 0.

Exercise 8 - Larger degree on top

The numerator degree 3 exceeds the denominator degree 2. Dividing by x^2 : $\frac{x+\frac{1}{x^2}}{2+\frac{1}{x}}$, whose numerator grows without bound.

Final answer: ∞ (no horizontal asymptote).

Exercise 9 - Toward negative infinity

Both degrees are 3. The ratio of leading coefficients is $\frac{2}{1}$. Dividing by x^3 : $\frac{2-\frac{1}{x^2}}{1+\frac{4}{x^3}} \rightarrow 2$.

Final answer: 2.

Exercise 10 - A square root

For large $|x|$, $\sqrt{x^2+1} \approx |x|$. As $x \rightarrow \infty$, $|x| = x$, so the ratio approaches $\frac{x}{x} = 1$.

As $x \rightarrow -\infty$, $|x| = -x$, so the ratio approaches $\frac{-x}{x} = -1$.

Final answer: 1 as $x \rightarrow \infty$ and -1 as $x \rightarrow -\infty$.

Exercise 11 - Horizontal asymptote and a table

The degrees are equal, so the limit is the ratio of leading coefficients: $\frac{6}{2} = 3$.

The table values 2.931, 2.9993, 3, 3 climb toward 3.

Final answer: $y = 3$.

Exercise 12 - Exponential and log limits

- a. $e^{-x} = \frac{1}{e^x} \rightarrow 0$.
- b. As x becomes very negative, e^x is a tiny positive number: 0
- c. The logarithm grows slowly but without bound: ∞
- d. Inputs near zero from the right have large negative logs: $-\infty$

Exercise 13 - A constant plus a fraction

As $x \rightarrow \infty$, $\frac{2}{x} \rightarrow 0$, so the expression approaches $5 - 0 = 5$.

Final answer: 5.

Exercise 14 - Squeeze at infinity

Since $-1 \leq \sin x \leq 1$, for $x > 0$ we have $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$.

Both bounds approach 0 as $x \rightarrow \infty$. Final answer: 0.

Exercise 15 - Two horizontal asymptotes

For $x > 0$, $\sqrt{x^2 + 4} \approx x$, so $g(x) \rightarrow 3$. For $x < 0$, $\sqrt{x^2 + 4} \approx -x$, so $g(x) \rightarrow -3$.

Final answer: $y = 3$ as $x \rightarrow \infty$ and $y = -3$ as $x \rightarrow -\infty$.

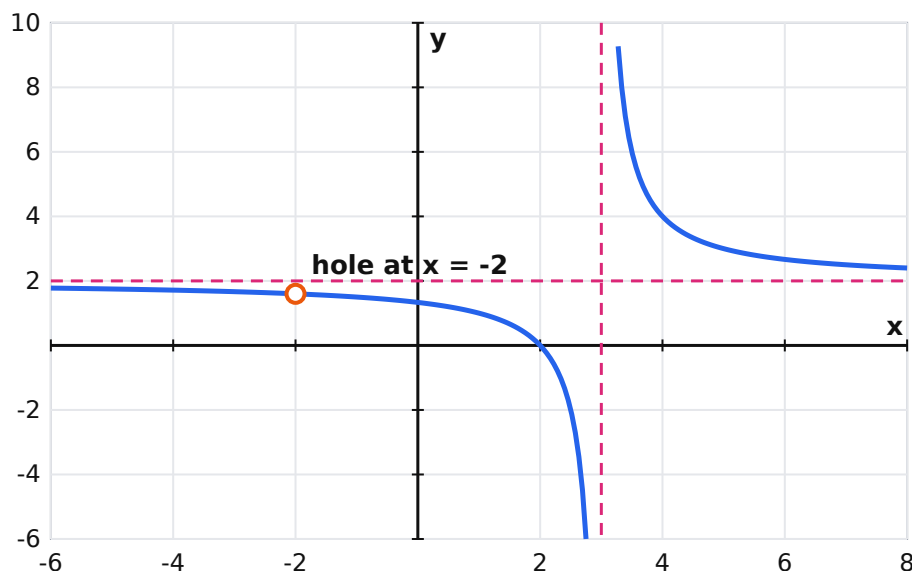
Exercise 16 - All asymptotes

Factor: $\frac{2(x-2)(x+2)}{(x-3)(x+2)} = \frac{2(x-2)}{x-3}$ for $x \neq -2$.

Vertical asymptote: $x = 3$ (numerator $2(1) = 2 \neq 0$). Hole: at $x = -2$, where $\frac{2(-4)}{-5} = \frac{8}{5}$.

Horizontal: equal degrees, $\frac{2}{1} = 2$.

Final answer: **VA $x = 3$, HA $y = 2$, hole at $(-2, 8/5)$.**



All asymptotes and the hole of f .

Exercise 17 - End behavior of a polynomial

For large $|x|$, the leading term $-2x^3$ dominates.

As $x \rightarrow \infty$, $-2x^3 \rightarrow -\infty$. As $x \rightarrow -\infty$, $-2x^3 \rightarrow \infty$.

Final answer: $\lim_{x \rightarrow \infty} p(x) = -\infty$ **and** $\lim_{x \rightarrow -\infty} p(x) = \infty$.

Exercise 18 - Medicine concentration

The numerator has degree 1 and the denominator degree 2, so $\frac{5t}{t^2+1} \rightarrow 0$.

Final answer: 0 mg/L. In the long run the medicine leaves the bloodstream, and the graph has the horizontal asymptote $C = 0$.

Exercise 19 - Average cost per item

Divide by n : $A(n) = 4 + \frac{200}{n}$. As $n \rightarrow \infty$, $\frac{200}{n} \rightarrow 0$.

Final answer: **\\$4 per sticker**. For huge orders the setup fee is spread thin, so the average cost approaches the \\$4 cost of each sticker.

Exercise 20 - Population limit

$$P(0) = \frac{1200}{1+3} = 300.$$

As $t \rightarrow \infty$, $e^{-0.5t} \rightarrow 0$, so $P(t) \rightarrow \frac{1200}{1+0} = 1200$.

Final answer: $P(0) = 300$ trout and a long-run population of 1200 trout.

Exercise 21 - Choosing a coefficient

The degrees are equal, so the horizontal asymptote is $y = \frac{a}{3}$.

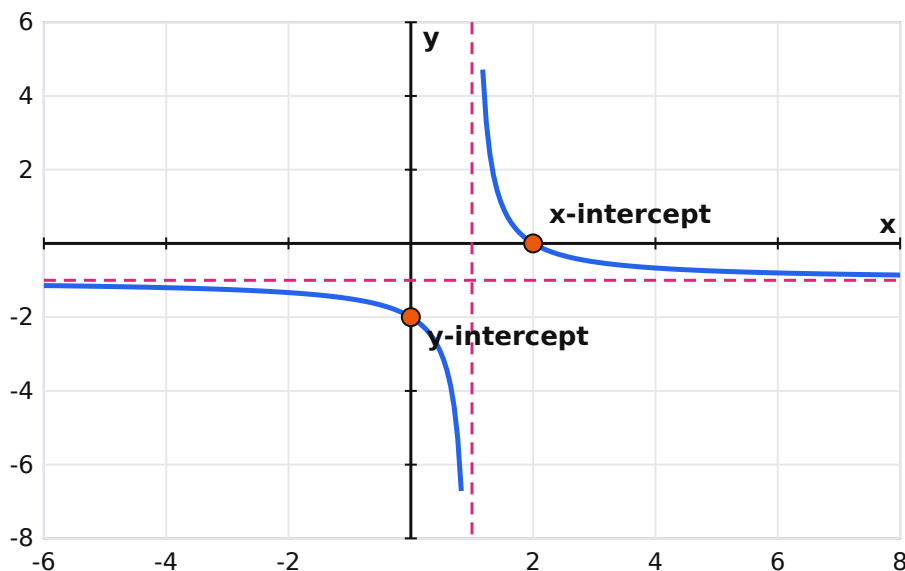
Set $\frac{a}{3} = 4$: $a = 12$. Final answer: $a = 12$.

Exercise 22 - Sketching with asymptotes

Vertical asymptote: $x = 1$. Horizontal: $\frac{-1}{1} = -1$, so $y = -1$.

The x-intercept is where $2 - x = 0$: $(2, 0)$. The y-intercept is $f(0) = \frac{2}{-1} = -2$: $(0, -2)$.

Final answer: **VA $x = 1$, HA $y = -1$, intercepts $(2, 0)$ and $(0, -2)$** ; one branch is above $y = -1$ to the left of $x = 1$ and the other below it to the right.



Graph of $f(x) = (2 - x)/(x - 1)$.