

AP Calculus AB - Infinite Limits and Asymptotes - Worksheet

Common Core AP CALC AB 1.13, AP CALC AB 1.14, AP CALC AB 1.15 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 22 problems practice infinite limits, vertical and horizontal asymptotes, and limits at infinity (AP Calculus AB, topics 1.13, 1.14, 1.15). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use square roots, exponential models, and real situations such as medicine concentration, average cost and a trout population. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.13, AP CALC AB 1.14, AP CALC AB 1.15**

Exercise 1 - One-sided infinite limits

Find $\lim_{x \rightarrow 3^+} \frac{1}{x-3}$ and $\lim_{x \rightarrow 3^-} \frac{1}{x-3}$. Does $\lim_{x \rightarrow 3} \frac{1}{x-3}$ exist?

Exercise 2 - An even power

Find $\lim_{x \rightarrow 2} \frac{1}{(x-2)^2}$.

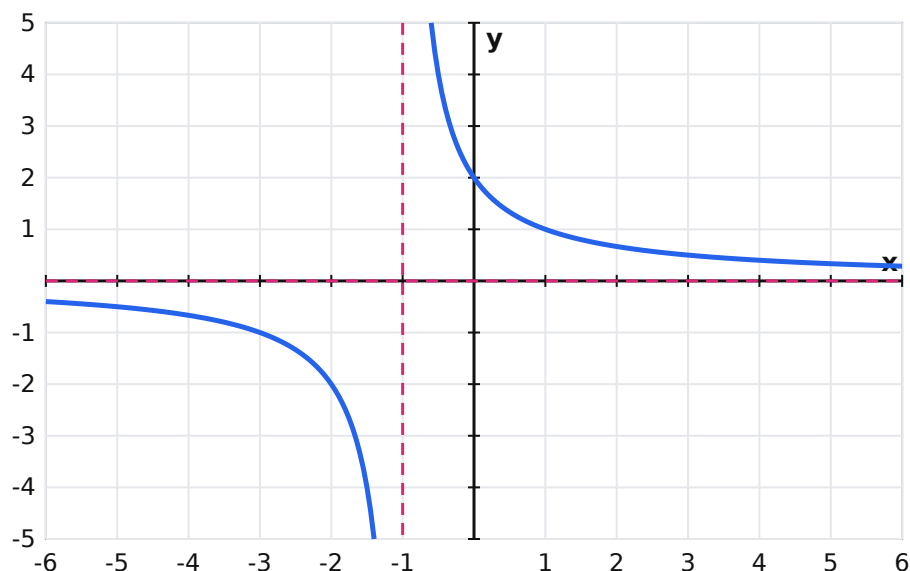
Exercise 3 - Sign analysis

Find $\lim_{x \rightarrow -1^-} \frac{x+3}{x+1}$ and $\lim_{x \rightarrow -1^+} \frac{x+3}{x+1}$.

Exercise 4 - Asymptote or hole?

Find all vertical asymptotes of $f(x) = \frac{x+2}{x^2-x-6}$ and identify any hole.

Exercise 5 - Asymptotes from a graph



The graph of $f(x) = 2/(x + 1)$.

The graph shows $f(x) = \frac{2}{x+1}$. State $\lim_{x \rightarrow -1^-} f(x)$, $\lim_{x \rightarrow -1^+} f(x)$, $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.

Exercise 6 - Equal degrees

Find $\lim_{x \rightarrow \infty} \frac{3x^2+2x}{5x^2-1}$.

Exercise 7 - Difference of roots at infinity

Find $\lim_{x \rightarrow \infty} (\sqrt{9x^2+1} - 3x)$.

Exercise 8 - Larger degree on top

Find $\lim_{x \rightarrow \infty} \frac{x^3+1}{2x^2+x}$.

Exercise 9 - Toward negative infinity

Find $\lim_{x \rightarrow -\infty} \frac{2x^3-x}{x^3+4}$.

Exercise 10 - A square root

Find $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}}{x+3}$ and $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+1}}{x+3}$.

Exercise 11 - Horizontal asymptote and a table

x	10	100	1,000	10,000
f(x)	2.931	2.9993	3	3

Values of $f(x)$ as x grows.

Find the horizontal asymptote of $f(x) = \frac{6x^2-5}{2x^2+3}$ and compare it with the table.

Exercise 12 - Exponential and log limits

Find (a) $\lim_{x \rightarrow \infty} e^{-x}$, (b) $\lim_{x \rightarrow -\infty} e^x$, (c) $\lim_{x \rightarrow \infty} \ln x$ and (d) $\lim_{x \rightarrow 0^+} \ln x$.

Exercise 13 - A constant plus a fraction

Find $\lim_{x \rightarrow \infty} (5 - \frac{2}{x})$.

Exercise 14 - Squeeze at infinity

Use the Squeeze Theorem to find $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$.

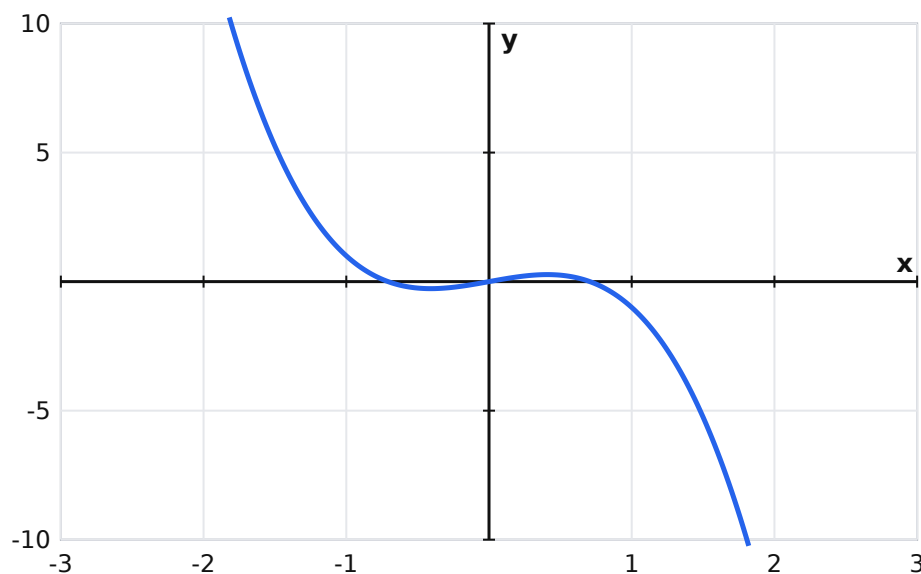
Exercise 15 - Two horizontal asymptotes

Find all horizontal asymptotes of $g(x) = \frac{3x}{\sqrt{x^2+4}}$.

Exercise 16 - All asymptotes

Find every vertical and horizontal asymptote and any hole for $f(x) = \frac{2x^2-8}{x^2-x-6}$.

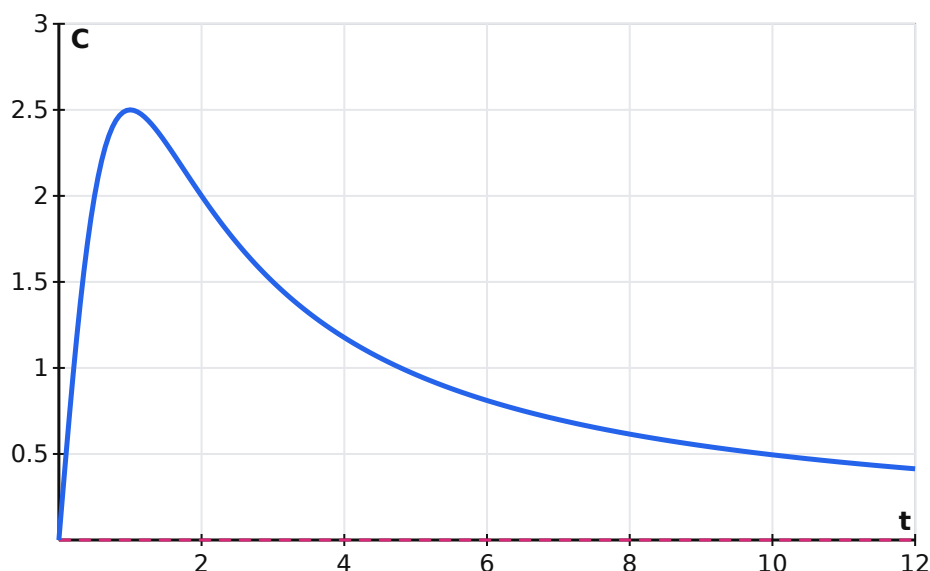
Exercise 17 - End behavior of a polynomial



The graph of $y = -2x^3 + x$.

Describe the end behavior of $p(x) = -2x^3 + x$ using limits.

Exercise 18 - Medicine concentration



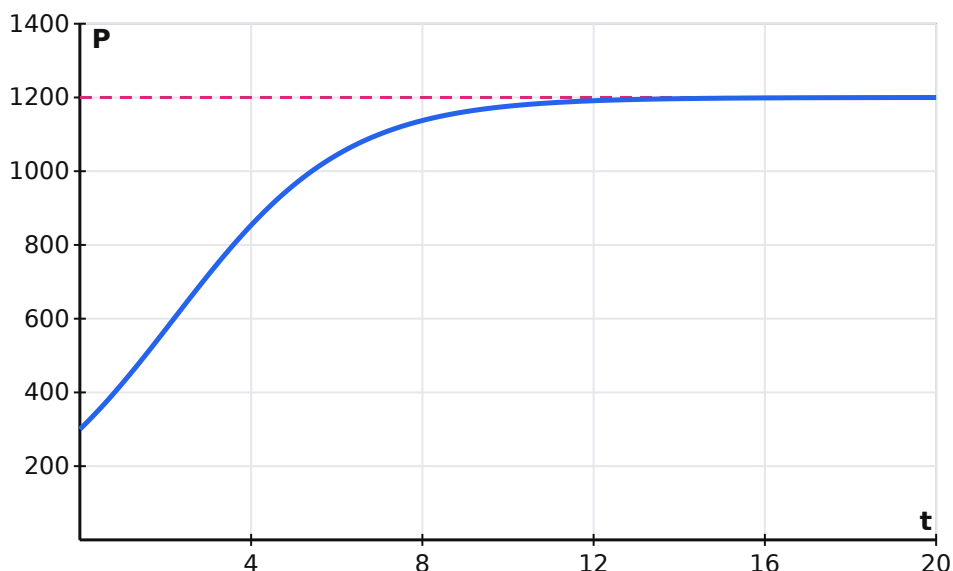
Medicine concentration over time.

Ethan takes a dose, and the concentration in his blood in mg/L after t hours is $C(t) = \frac{5t}{t^2+1}$. Find $\lim_{t \rightarrow \infty} C(t)$ and explain what it means.

Exercise 19 - Average cost per item

A school store spends \\$200 to set up a custom-sticker order plus \\$4 per sticker. The average cost per sticker is $A(n) = \frac{4n+200}{n}$ dollars for n stickers. Find $\lim_{n \rightarrow \infty} A(n)$ and interpret it.

Exercise 20 - Population limit



Population growth that levels off.

The number of trout in a pond after t years is $P(t) = \frac{1200}{1 + 3e^{-0.5t}}$. Find $P(0)$ and $\lim_{t \rightarrow \infty} P(t)$.

Exercise 21 - Choosing a coefficient

Find the value of a so that the graph of $f(x) = \frac{ax^2 + x}{3x^2 - 2}$ has the horizontal asymptote $y = 4$.

Exercise 22 - Sketching with asymptotes

For $f(x) = \frac{2-x}{x-1}$, find the asymptotes and both intercepts, and describe the graph.