

AP Calculus AB - The Chain Rule - Lesson

Common Core AP CALC AB 3.1 · maths-pdf.fr/en/

The chain rule differentiates a composite function: if $y = f(g(x))$, then $dy/dx = f'(g(x)) \cdot g'(x)$. Differentiate the outer function while leaving the inner function unchanged, then multiply by the derivative of the inner function. For example, the derivative of $(x^2 + 1)^3$ is $3(x^2 + 1)^2 \cdot 2x$.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.1**

Key facts

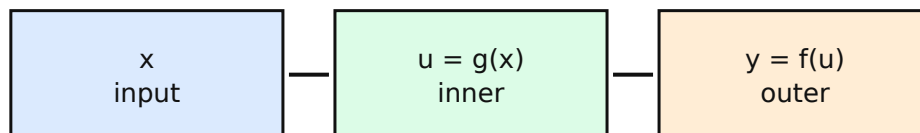
- Chain rule: $dy/dx = f'(g(x)) \cdot g'(x)$, or dy/du times du/dx .
- Outer derivative first, inner function unchanged, then multiply by the inner derivative.
- $d/dx e^{g(x)} = e^{g(x)} g'(x)$ and $d/dx \ln(g(x)) = g'(x)/g(x)$.
- Functions with three layers need the chain rule twice.

What is the chain rule?

Many functions are built by putting one function inside another, such as $(3x+2)^5$ or $\sin(x^2)$. These are *composite functions*. The chain rule gives their derivative.

Chain rule

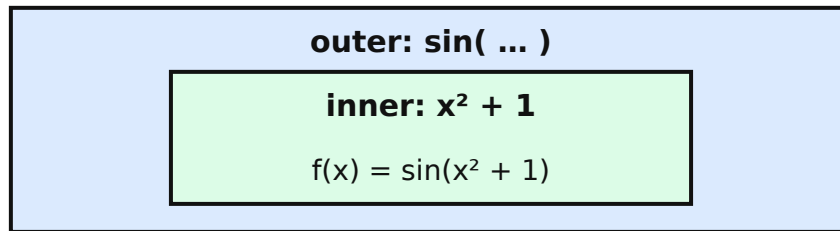
If $y = f(g(x))$, then $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$. Equivalently, with $u = g(x)$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.



$dy/dx = (dy/du) \cdot (du/dx)$: multiply the rates along the chain.

Think of it as multiplying rates. If y changes 4 times as fast as u and u changes 3 times as fast as x , then y changes 12 times as fast as x .

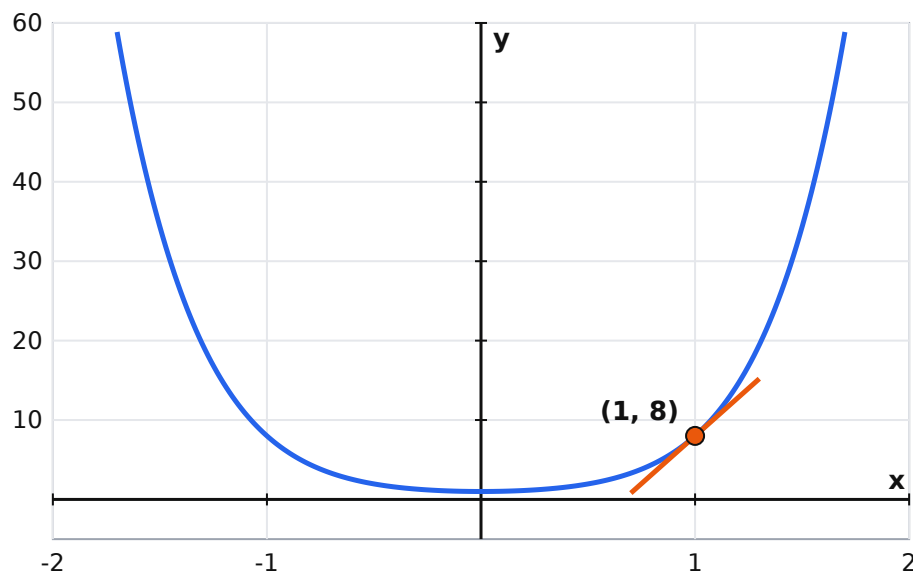
How do you apply the chain rule step by step?



Peel the layers from the outside: differentiate sine, keep the inside, then multiply by the derivative of the inside.

1. Identify the outer function and the inner function u .
2. Differentiate the outer function and leave the inner function unchanged.
3. Multiply by the derivative of the inner function.
4. Simplify, and substitute a given value of x last.

Example. For $f(x) = (x^2 + 1)^3$, the outer function is u^3 and the inner function is $u = x^2 + 1$. So $f'(x) = 3(x^2 + 1)^2 \cdot 2x = 6x(x^2 + 1)^2$. At $x = 1$ the slope is $6 \cdot 4 = 24$.

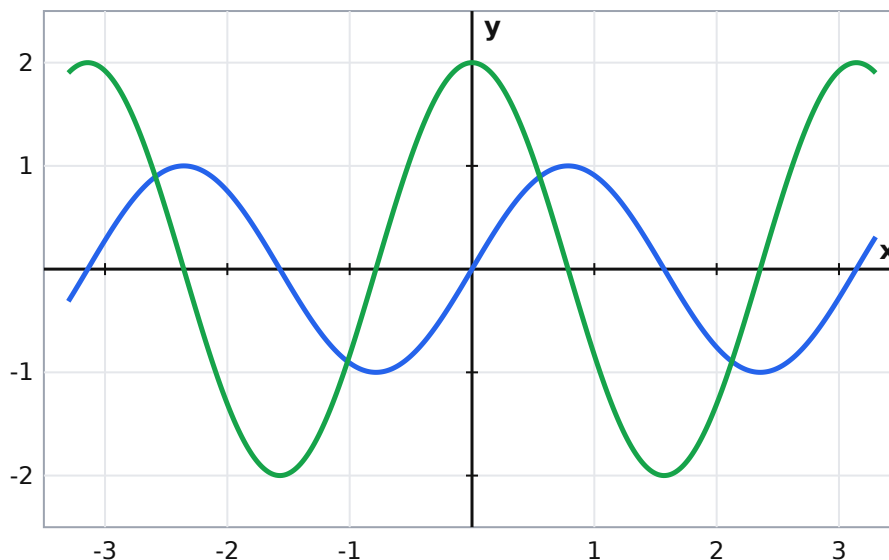


The slope at $x = 1$ is $6x(x^2 + 1)^2 = 24$.

What are the most common chain rule patterns?

Function	Derivative
$[g(x)]^n$	$n[g(x)]^{n-1} g'(x)$
$\sin(g(x))$	$\cos(g(x)) g'(x)$
$e^{g(x)}$	$e^{g(x)} g'(x)$
$\ln(g(x))$	$\frac{g'(x)}{g(x)}$

Function	Derivative
$\sqrt{g(x)}$	$\frac{g'(x)}{2\sqrt{g(x)}}$



The derivative of $\sin 2x$ is $2\cos 2x$: the factor 2 comes from the inner function $2x$.

A function can have three or more layers. For $\cos^3(2x)$, peel from the outside: the cube, then cosine, then $2x$. The derivative is $3\cos^2(2x) \cdot (-\sin(2x)) \cdot 2 = -6\cos^2(2x)\sin(2x)$.

The most common mistake is forgetting the factor $g'(x)$ from the inner function. A quick check: if the inside is not just x , there must be an extra factor.

How do you use the chain rule with a table of values?

Exams often give numbers instead of formulas. Suppose $g(1)=3$, $g'(1)=2$ and $f'(3)=-4$. For $h(x)=f(g(x))$, the chain rule gives $h'(1)=f'(g(1)) \cdot g'(1)=f'(3) \cdot 2=-8$. Notice that the outer derivative is evaluated at $g(1)$, not at 1. Writing that evaluation point explicitly prevents the most common table error.

The same idea explains real rates. If a balloon's radius grows with time and its volume grows with radius, the volume's rate with respect to time is the product of the two rates.

Questions about the chain rule

What is a composite function?

A composite function applies one function to the output of another, written $f(g(x))$. For example, $\sin(x^2)$ applies sine to x squared.

How do you know when to use the chain rule?

Use it whenever the input of a function is something more complicated than just x , such as $3x + 2$, x^2 or e^x . If you can name an outer and an inner function, apply the chain rule.

What is the derivative of $e^{(3x)}$?

The derivative is $3e^{(3x)}$. The outer derivative $e^{(3x)}$ is multiplied by the inner derivative 3 .

What is the derivative of $\ln(g(x))$?

It is $g'(x)$ divided by $g(x)$. For example, the derivative of $\ln(x^2 + 1)$ is $2x/(x^2 + 1)$.

How do you differentiate with three layers?

Work from the outside in. Differentiate the outer layer, then the middle layer, then the inner layer, and multiply all the factors together.