

AP Calculus AB - The Chain Rule - Answer Key

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This answer key gives a full worked solution for every problem in the chain rule worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.1**

Exercise 1 - Power of a linear function

Outer function: u^5 . Inner function: $u = 3x + 2$.

Chain rule: $f'(x) = 5(3x + 2)^4 \cdot 3$.

Final answer: $f'(x) = 15(3x + 2)^4$.

Exercise 2 - Power of a quadratic

Outer function: u^3 . Inner function: $u = x^2 - 4x$.

Chain rule: $f'(x) = 3(x^2 - 4x)^2 \cdot (2x - 4)$.

Final answer: $f'(x) = 3(2x - 4)(x^2 - 4x)^2$.

Exercise 3 - Square root of a linear function

Outer function: $\sqrt{u}$. Inner function: $u = 5x - 1$.

Chain rule: $f'(x) = \frac{1}{2\sqrt{5x-1}} \cdot 5$.

Final answer: $f'(x) = \frac{5}{2\sqrt{5x-1}}$.

Exercise 4 - Reciprocal power

Rewrite: $f(x) = (2x + 3)^{-2}$.

Outer function: u^{-2} . Inner function: $u = 2x + 3$.

Chain rule: $f'(x) = -2(2x + 3)^{-3} \cdot 2$.

Final answer: $f'(x) = -\frac{4}{(2x + 3)^3}$.

Exercise 5 - Sine of a multiple

Outer function: $\sin u$. Inner function: $u = 4x$.

Chain rule: $f'(x) = \cos(4x) \cdot 4$.

Final answer: $f'(x) = 4\cos(4x)$.

Exercise 6 - Cosine of a power

Outer function: $\cos u$. Inner function: $u = x^2$.

Chain rule: $f'(x) = -\sin(x^2) \cdot 2x$.

Final answer: $f'(x) = -2x\sin(x^2)$.

Exercise 7 - Exponential with a multiple

Outer function: e^u . Inner function: $u = 3x$.

Chain rule: $f'(x) = e^{3x} \cdot 3$.

Final answer: $f'(x) = 3e^{3x}$.

Exercise 8 - Bell curve

Outer function: e^u . Inner function: $u = -x^2$.

Chain rule: $f'(x) = e^{-x^2} \cdot (-2x)$.

Final answer: $f'(x) = -2xe^{-x^2}$.

At $x = 0$ the factor $-2x$ is 0, which matches the peak of the bell curve.

Exercise 9 - Log of a linear function

Outer function: $\ln u$. Inner function: $u = 5x + 2$.

Chain rule: $f'(x) = \frac{1}{5x+2} \cdot 5$.

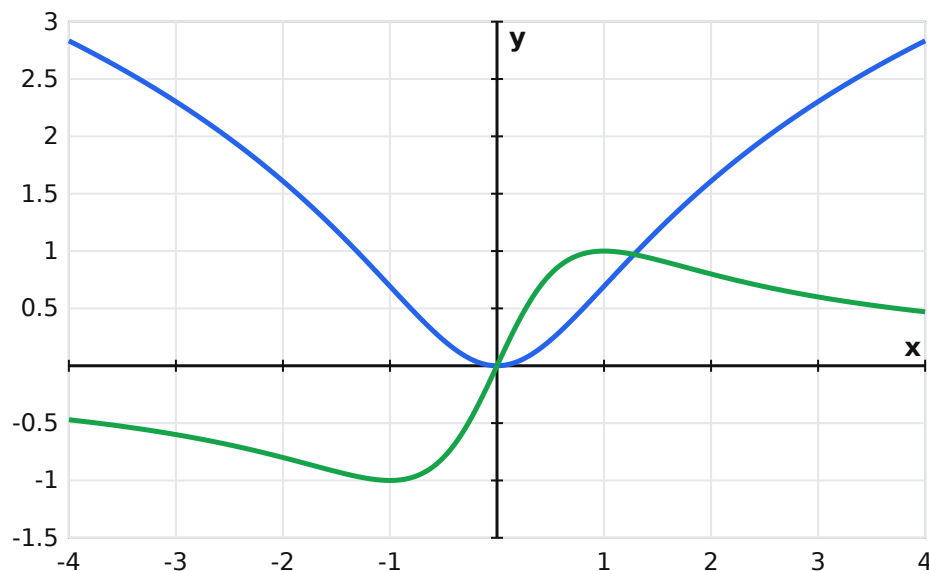
Final answer: $f'(x) = \frac{5}{5x+2}$.

Exercise 10 - Log of a quadratic

Outer function: $\ln u$. Inner function: $u = x^2 + 1$.

Chain rule: $f'(x) = \frac{1}{x^2+1} \cdot 2x$.

Final answer: $f'(x) = \frac{2x}{x^2+1}$.



$\ln(x^2 + 1)$ and its derivative.

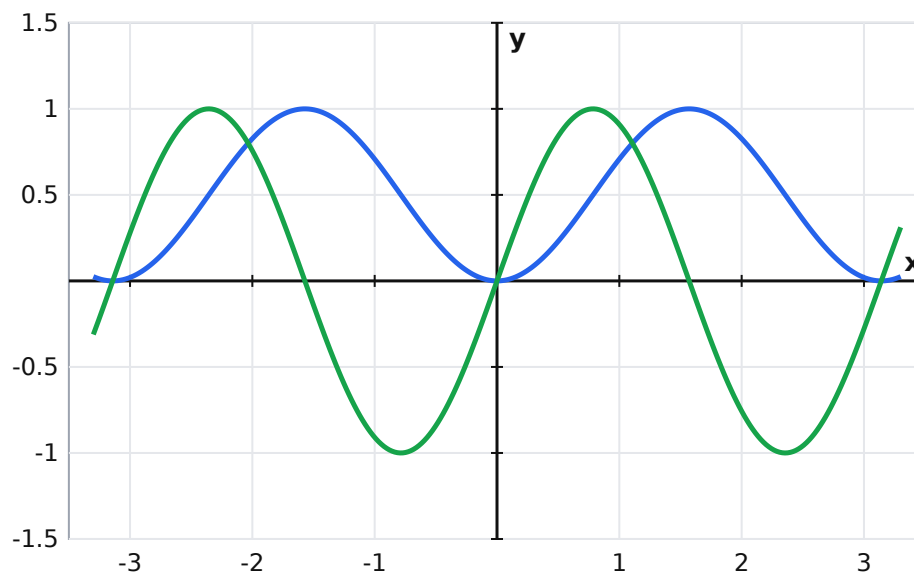
Exercise 11 - Sine squared

Outer function: u^2 . Inner function: $u = \sin x$.

Chain rule: $f'(x) = 2\sin x \cdot \cos x$.

Final answer: $f'(x) = 2\sin(x)\cos(x)$.

This equals $\sin 2x$ by a double-angle identity.



$\sin^2 x$ and its derivative $\sin 2x$.

Exercise 12 - Tangent of a multiple

Outer function: $\tan u$. Inner function: $u = 3x$.

Chain rule: $f'(x) = \sec^2(3x) \cdot 3$.

Final answer: $f'(x) = 3\sec^2(3x)$.

Exercise 13 - Sine of an exponential

For f : outer $\sin u$, inner e^x , so $f'(x) = \cos(e^x) \cdot e^x$.

For g : outer e^u , inner $\sin x$, so $g'(x) = e^{\sin x} \cos x$.

Final answer: $f'(x) = e^x \cos(e^x)$ and $g'(x) = e^{\sin x} \cos x$.

Exercise 14 - Three layers

Layers: cube, cosine, then $2x$. Work from the outside:

$f'(x) = 3\cos^2(2x) \cdot (-\sin 2x) \cdot 2$.

Final answer: $-6\cos^2(2x)\sin(2x)$.

Exercise 15 - Chain inside a product

Product rule with $u = x^2$ and $v = \sqrt{x^2+1}$. By the chain rule, $v' = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$.

$$f'(x) = 2x\sqrt{x^2+1} + \frac{x^3}{\sqrt{x^2+1}} = \frac{2x(x^2+1) + x^3}{\sqrt{x^2+1}}.$$

Final answer: $\frac{3x^3+2x}{\sqrt{x^2+1}}$.

Exercise 16 - Chain inside a quotient

Quotient rule with $u = e^{2x}$, $u' = 2e^{2x}$ (chain rule), $v = x+1$, $v' = 1$.

$$f'(x) = \frac{2e^{2x}(x+1) - e^{2x}}{(x+1)^2} = \frac{e^{2x}(2x+1)}{(x+1)^2}.$$

Final answer: $\frac{e^{2x}(2x+1)}{(x+1)^2}$.

Exercise 17 - Composite from a table

Chain rule: $h'(1) = f'(g(1)) \cdot g'(1) = f'(3) \cdot 2 = (-4)(2) = -8$.

Final answer: -8 .

function	value at point	derivative
g at $x = 1$	3	2
f at $u = 3$	5	-4

Multiply the derivatives along the chain: $(-4)(2) = -8$.

Exercise 18 - Tangent line with the chain rule

$y(1) = 2^3 = 8$. $y' = 3(x^2+1)^2 \cdot 2x = 6x(x^2+1)^2$, so $y'(1) = 6 \cdot 4 = 24$.

Tangent: $y - 8 = 24(x - 1)$. Final answer: $y = 24x - 16$.

Exercise 19 - Horizontal tangents

Product and chain rules: $f'(x) = e^{-x^2/2} + x \cdot e^{-x^2/2} \cdot (-x) = e^{-x^2/2}(1 - x^2)$.

Since the exponential is never zero, set $1-x^2=0$.

Final answer: $x=1$ and $x=-1$.

Exercise 20 - Cooling coffee

$$T'(t) = 110 \cdot e^{-0.05t} \cdot (-0.05) = -5.5e^{-0.05t}.$$

$$T'(10) = -5.5e^{-0.5} \approx -3.34.$$

Final answer: $T'(10) \approx -3.34$ $^{\circ}\text{F}/\text{min}$: after 10 minutes the coffee is cooling at about 3.34 degrees per minute.

Exercise 21 - Pendulum

$$\theta'(t) = 0.3 \cdot (-\sin 2t) \cdot 2 = -0.6 \sin 2t.$$

$$\text{At } t = \frac{\pi}{4}: -0.6 \sin \frac{\pi}{2} = -0.6.$$

Final answer: -0.6 rad/s.

Exercise 22 - Nested radicals

Outer: $\sqrt{u}$ with $u = x + \sqrt{x}$. Inner derivative: $1 + \frac{1}{2\sqrt{x}}$.

$$f'(x) = \frac{1}{2\sqrt{x+\sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}} \right).$$

$$\text{Final answer: } \frac{1}{2\sqrt{x+\sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}} \right).$$