

AP Calculus AB - The Chain Rule - Test Answer Key

Common Core AP CALC AB 3.1 · maths-pdf.fr/en/

Use this answer key to score the chain rule test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.1**

Question 1 - Power chain [2 pts]

$$f'(x) = 6(4x-7)^5 \cdot 4. \text{ Final answer: } 24(4x-7)^5.$$

Scoring: 1 point for the outer derivative; 1 point for the inner factor 4.

Question 2 - Trig chain [2 pts]

$$g'(x) = \cos(x^3) \cdot 3x^2 - \sin(5x) \cdot 5.$$

$$\text{Final answer: } 3x^2 \cos(x^3) - 5 \sin(5x).$$

Scoring: 1 point per term.

Question 3 - Exponential and log chain [2 pts]

$$h'(x) = e^{x^2-3x}(2x-3) + \frac{4}{4x+1}.$$

$$\text{Final answer: } (2x-3)e^{x^2-3x} + \frac{4}{4x+1}.$$

Scoring: 1 point per term.

Question 4 - Three layers [3 pts]

Layers: cube, sine, then x^2+1 .

$$k'(x) = 3\sin^2(x^2+1) \cdot \cos(x^2+1) \cdot 2x.$$

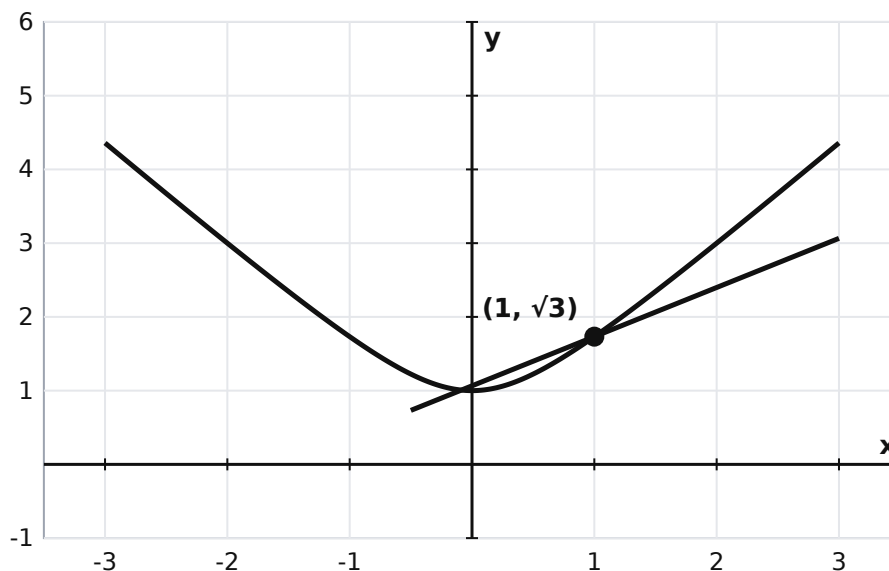
$$\text{Final answer: } 6x\sin^2(x^2+1)\cos(x^2+1).$$

Scoring: 1 point for each layer correctly differentiated.

Question 5 - Tangent line [3 pts]

$$y(1) = \sqrt{3}. \quad y' = \frac{4x}{2\sqrt{2x^2+1}} = \frac{2x}{\sqrt{2x^2+1}}, \text{ so } y'(1) = \frac{2}{\sqrt{3}}.$$

$$\text{Final answer: } y - \sqrt{3} = \frac{2}{\sqrt{3}}(x - 1).$$



Graph of $y = \sqrt{2x^2 + 1}$.

Scoring: 1 point for $y(1)$; 1 point for $y'(1)$; 1 point for the equation.

Question 6 - Composite from a table [3 pts]

$$\text{a. } f'(g(1)) \cdot g'(1) = f'(2) \cdot (-5) = 3(-5) = -15.$$

$$\text{b. } p'(4) = 2f(4)f'(4) = 2(7)(6) = 84.$$

Scoring: (a) 1 point for $g(1) = 2$ and 1 point for the product; (b) 1 point.

Question 7 - Product and chain [2 pts]

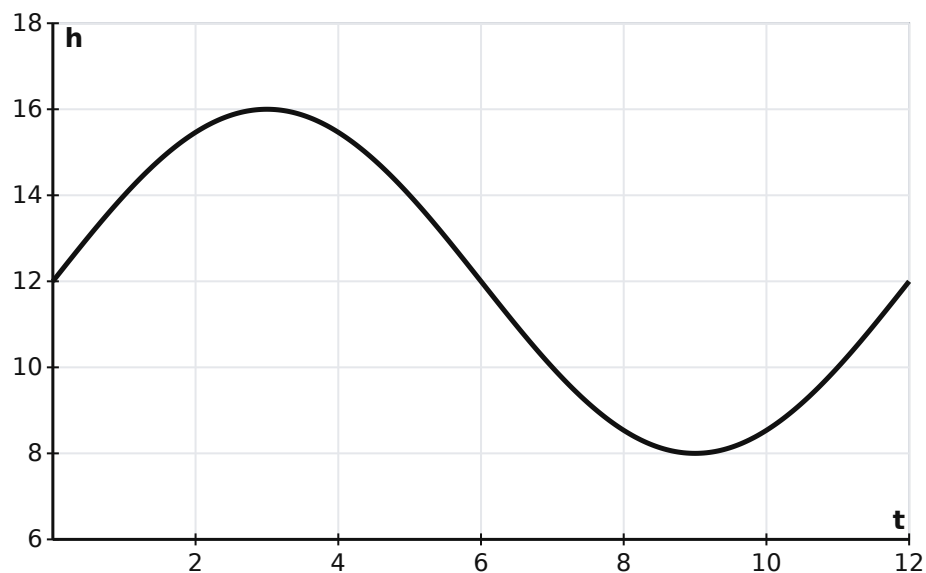
$$p'(x) = 1 \cdot e^{3x} + x \cdot 3e^{3x} = e^{3x}(1 + 3x).$$

Final answer: $e^{3x}(1+3x)$.

Scoring: 1 point for the product rule; 1 point for the chain factor 3.

Question 8 - Tide height [3 pts]

- a. $h'(t) = 4 \cos\left(\frac{\pi t}{6}\right) \cdot \frac{\pi}{6} = \frac{2\pi}{3} \cos\left(\frac{\pi t}{6}\right)$.
- b. $h'(0) = \frac{2\pi}{3} \approx 2.09$ ft/h and $h'(3) = \frac{2\pi}{3} \cos\frac{\pi}{2} = 0$.
- c. At 3 hours the tide is at its highest point (16 feet), so it is momentarily neither rising nor falling.



Tide height in feet.

Scoring: 1 point per part.

Total

20 points