

AP Calculus AB - Concavity and the Second Derivative - Lesson

Common Core AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9 · maths-pdf.fr/en/

The second derivative f'' shows concavity: f is concave up where f'' is positive and concave down where f'' is negative. An inflection point is where concavity changes, so f'' changes sign. The Second Derivative Test classifies a critical number c with $f'(c) = 0$: a positive $f''(c)$ gives a relative minimum and a negative $f''(c)$ gives a relative maximum.

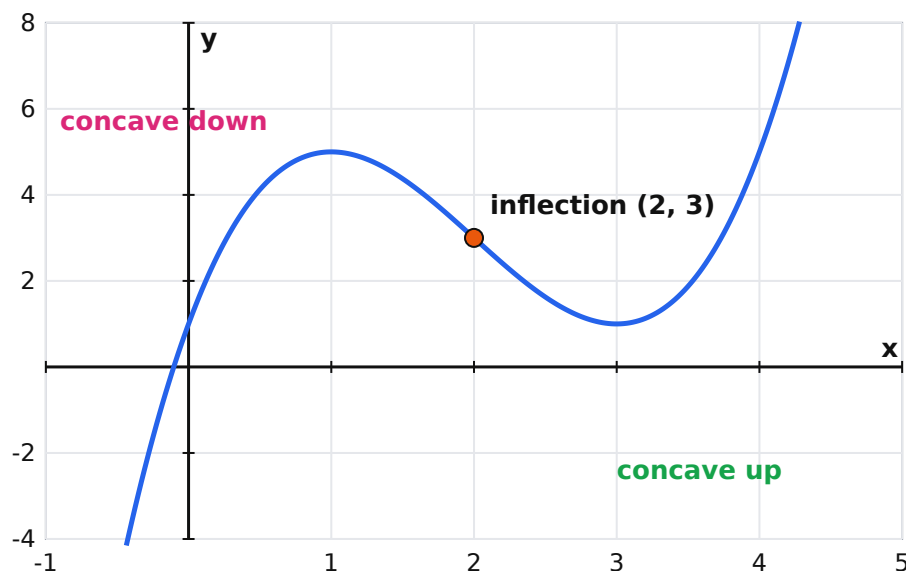
AP Calculus AB · Common Core State Standards: **AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9**

Key facts

- A positive f'' means concave up (slope increasing); a negative f'' means concave down (slope decreasing).
- An inflection point needs a change in the sign of f'' , not just $f'' = 0$.
- Second Derivative Test: $f'(c) = 0$ and a positive $f''(c)$ gives a relative minimum; a negative $f''(c)$ gives a maximum.
- If $f''(c) = 0$ the test fails; use the First Derivative Test.

What does the second derivative tell you?

The second derivative f'' measures how the slope changes. If $f'' > 0$, the slope is increasing and the graph is **concave up**, like a cup. If $f'' < 0$, the slope is decreasing and the graph is **concave down**, like a frown.

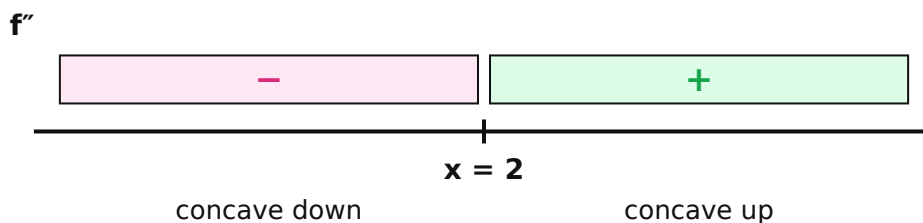


The graph bends down before $x = 2$ and bends up after it; $(2, 3)$ is an inflection point.

Inflection point

An **inflection point** is a point on the graph where the concavity changes. At such a point $f'' = 0$ or f'' is undefined, and f'' changes sign. A zero of f'' alone is not enough.

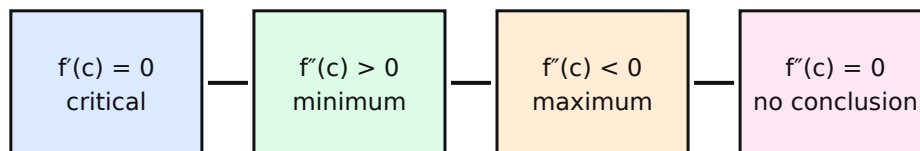
Example. For $f(x) = x^3 - 6x^2 + 9x + 1$, $f''(x) = 6x - 12$, which is zero at $x = 2$ and changes sign there. The graph is concave down on $(-\infty, 2)$, concave up on $(2, \infty)$, with an inflection point at $(2, 3)$.



A sign chart of f'' shows where the graph is concave up or down.

What is the second derivative test?

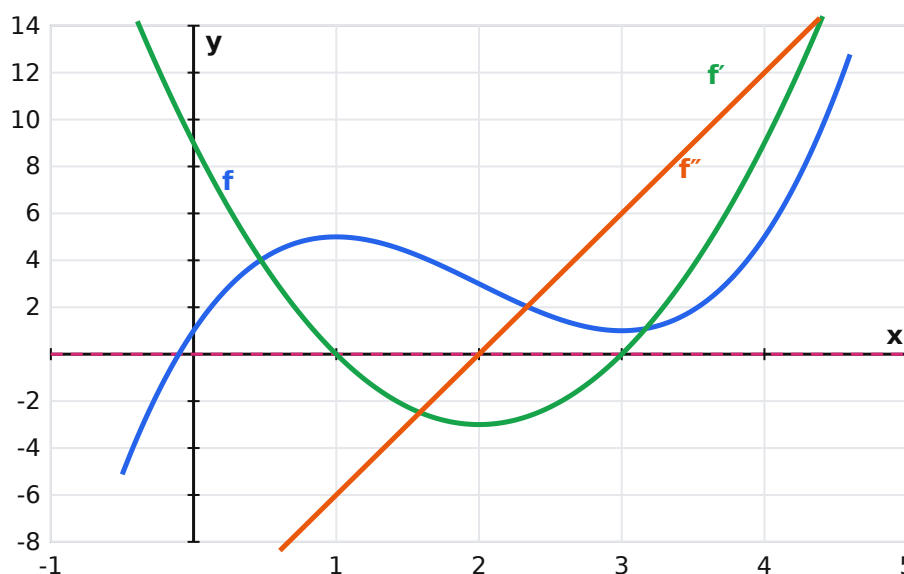
If $f'(c) = 0$, check $f''(c)$. A positive value means the graph is a cup at c , so $f(c)$ is a relative minimum. A negative value gives a relative maximum. If $f''(c) = 0$, the test fails and you must use the First Derivative Test.



The second derivative test classifies a critical number with one evaluation.

How do f , f' and f'' connect?

Feature of f	Where to read it
Increasing or decreasing	$f' > 0$ or $f' < 0$
Relative extremum	f' changes sign
Concave up or down	$f'' > 0$ or $f'' < 0$ (or f' increasing or decreasing)
Inflection point	f'' changes sign (f' has a local extremum)



f (blue), f' (green) and f'' (orange) for the same function.

Example (second derivative test). For $f(x) = x^3 - 12x + 3$, $f' = 3x^2 - 12 = 0$ gives $x = \pm 2$, and $f'' = 6x$. Since $f''(2) = 12$ is positive, $x = 2$ is a relative minimum. Since $f''(-2) = -12$ is negative, $x = -2$ is a relative maximum. No sign chart is needed, which makes this test fast when f'' is simple.

Look at the green graph of f' : it has its lowest point at $x = 2$, exactly where the blue graph of f changes concavity. An inflection point of f is a place where the slope is at a local extreme.

Questions about concavity and the second derivative

What does concave up mean?

A graph is concave up on an interval if its slope is increasing there, which means f'' is positive. The graph bends like a cup and lies above its tangent lines.

What is an inflection point?

It is a point where the graph changes concavity. The second derivative is zero or undefined there and changes sign.

Does $f''(c) = 0$ always mean an inflection point?

No. For $f(x) = x^4$, $f''(0) = 0$ but f'' is positive on both sides, so the concavity never changes and there is no inflection point.

How does the second derivative test work?

At a critical number c where $f'(c) = 0$, a positive $f''(c)$ means relative minimum and a negative $f''(c)$ means relative maximum. If $f''(c) = 0$, the test is inconclusive.

How can you find concavity from the graph of f' ?

f is concave up where the graph of f' is increasing and concave down where the graph of f' is decreasing. Inflection points of f occur at local extrema of f' .