

AP Calculus AB - Concavity and the Second Derivative - Worksheet

Common Core AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9 · maths-pdf.fr/en/

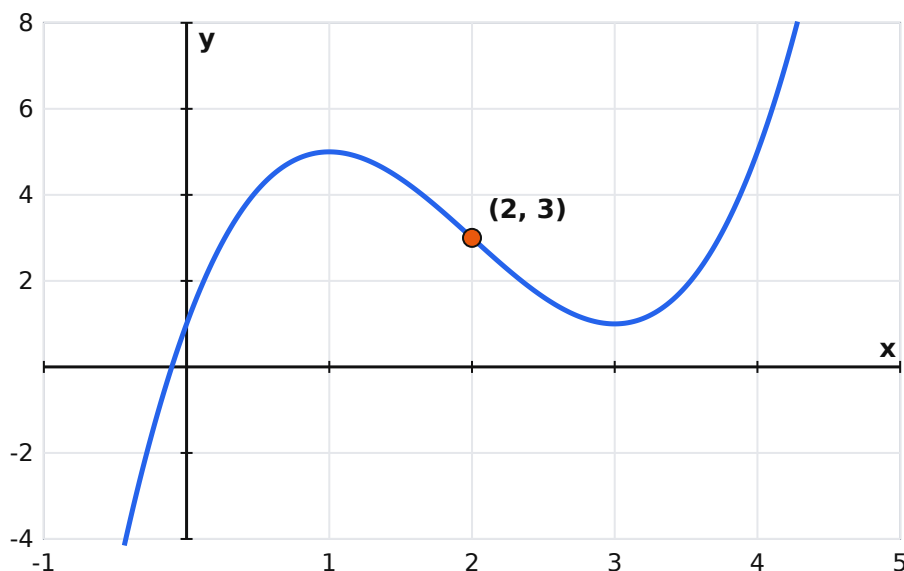
Name: _____ Class: _____ Date: ____/____/____

These 21 problems practice concavity, inflection points, the Second Derivative Test and connecting the graphs of f , f' and f'' (AP Calculus AB, topics 5.7, 5.8, 5.9). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems connect graphs of f and its derivatives and use the second derivative in situations such as sales growth and marginal cost. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9**

Exercise 1 - Concavity of a cubic



The graph of f .

Find the intervals of concavity and the inflection point of $f(x) = x^3 - 6x^2 + 9x + 1$.

Exercise 2 - Concavity of a logarithm

Find the intervals of concavity and the inflection points of $f(x) = \ln(x^2+1)$.

Exercise 3 - Exponential inflection

Find the inflection point of $f(x) = xe^{-x}$.

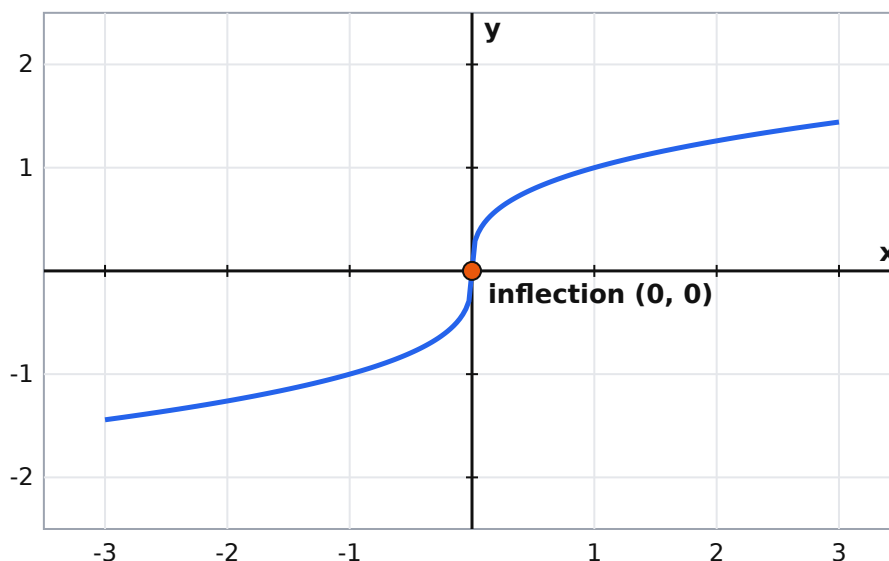
Exercise 4 - Always concave down

Show that $f(x) = \ln x$ has no inflection point.

Exercise 5 - Sine on an interval

Find the concavity and inflection points of $f(x) = \sin x$ on $(0, 2\pi)$.

Exercise 6 - Inflection where f double prime is undefined



The graph of $f(x) = \sqrt[3]{x}$.

Show that $f(x) = \sqrt[3]{x}$ has an inflection point at $x=0$, even though $f''(0)$ does not exist.

Exercise 7 - The bell curve

Find the inflection points of $f(x) = \frac{1}{x^2+1}$.

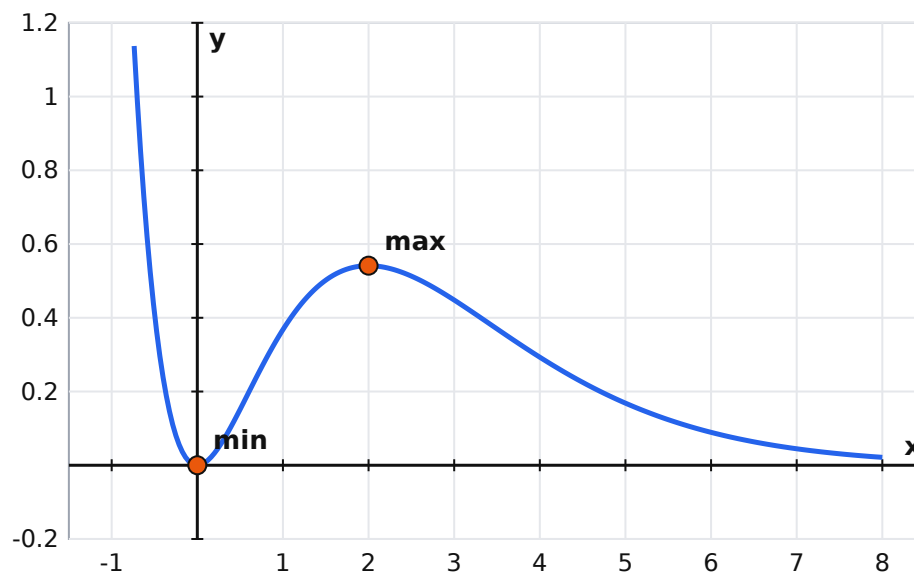
Exercise 8 - Zero but not an inflection

Does $f(x) = x^4$ have an inflection point at $x=0$?

Exercise 9 - Second derivative test, cubic

Use the Second Derivative Test on $f(x) = x^3 - 12x + 3$.

Exercise 10 - Second derivative test, exponential



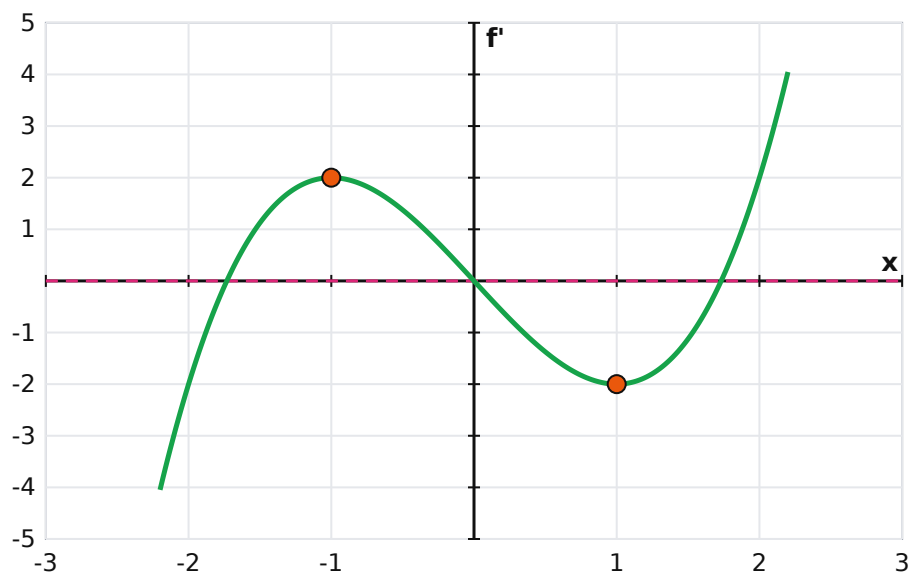
The graph of $f(x) = x^2e^{-x}$.

Use the Second Derivative Test to classify the critical numbers of $f(x) = x^2e^{-x}$.

Exercise 11 - A test that fails

Show that the Second Derivative Test fails for $f(x) = x^4$ at $x = 0$, and use another test to classify it.

Exercise 12 - Concavity from the graph of f prime



The graph of $f'(x) = x^3 - 3x$.

The graph shows $f'(x) = x^3 - 3x$. On which intervals is f concave up? Where are the inflection points of f ?

Exercise 13 - Conclusions from a sign table

interval	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
sign of f''	–	+	–

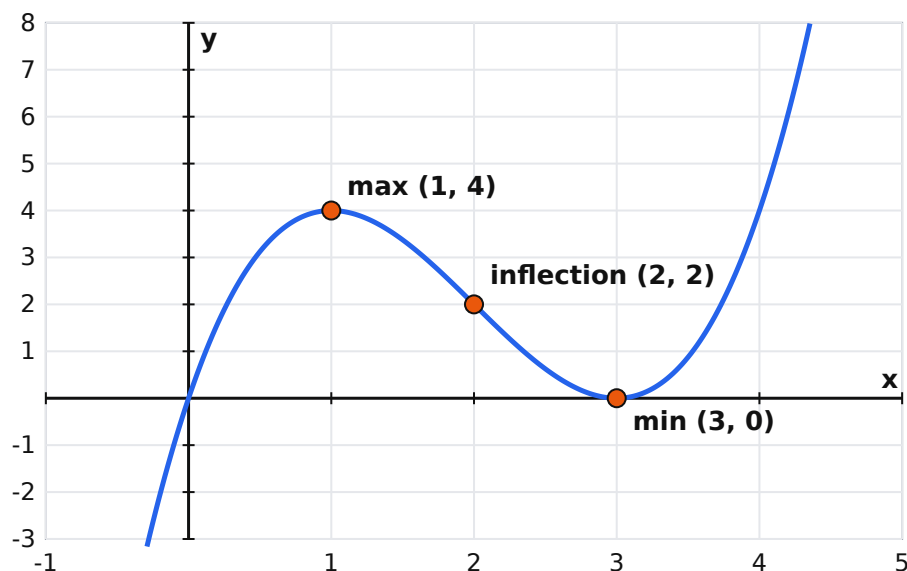
Signs of f'' for a continuous function f .

Describe the concavity of f and the inflection points.

Exercise 14 - All features of a cubic

List the intervals of increase and decrease, relative extrema, concavity and inflection point of $f(x) = x^3 - 3x^2$.

Exercise 15 - Curve sketch



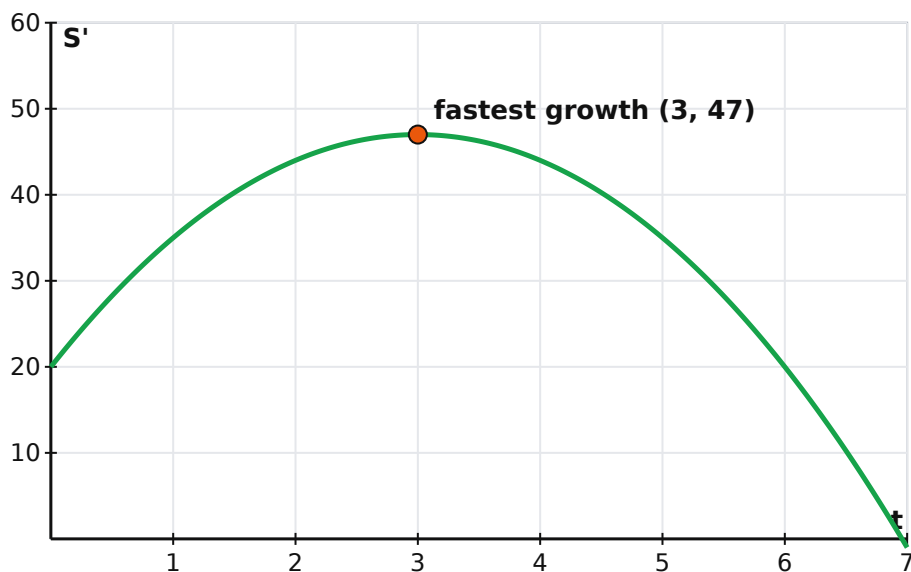
The graph of $f(x) = x^3 - 6x^2 + 9x$.

Find the intercepts, relative extrema and inflection point of $f(x) = x^3 - 6x^2 + 9x$.

Exercise 16 - Finding coefficients

The graph of $f(x) = x^3 + ax^2 + bx$ has an inflection point at $x = 2$ and a horizontal tangent at $x = 1$. Find a and b .

Exercise 17 - Growth of sales

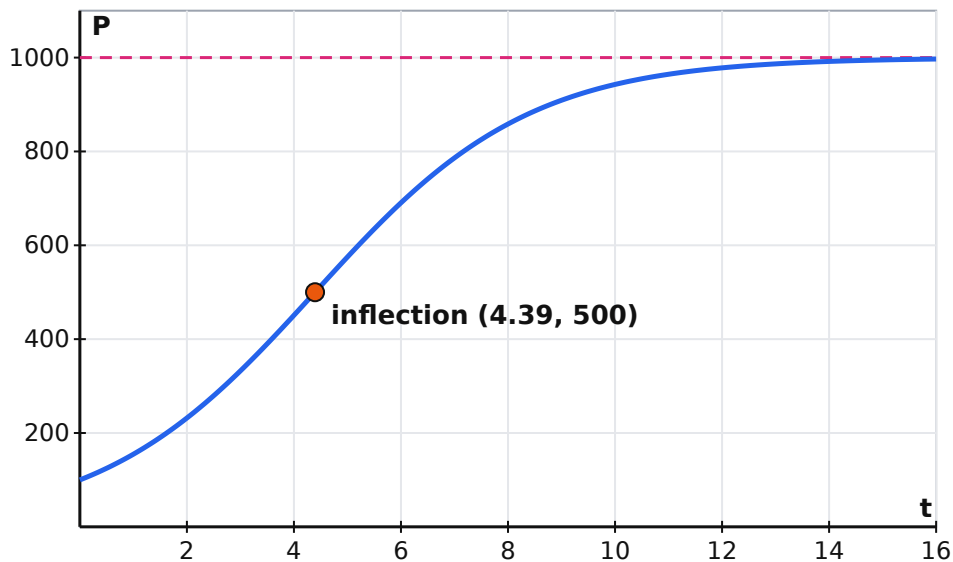


The sales rate $S'(t)$ peaks at $t = 3$.

A new game's total sales in thousands of dollars after t months is $S(t) = -t^3 + 9t^2 + 20t$. When is

the sales rate the greatest, and what is it?

Exercise 18 - Logistic population



A logistic population curve.

A bird population is $P(t) = \frac{1000}{1+9e^{-t/2}}$ after t years. At what time does the population grow fastest? What is the population then?

Exercise 19 - Marginal cost

A shop's cost in dollars to make x mugs is $C(x) = 0.01x^3 - 0.9x^2 + 30x + 500$. Find the number of mugs at which the marginal cost $C'(x)$ is smallest, and that minimum marginal cost.

Exercise 20 - Both tests together

Classify the critical numbers of $f(x) = x^5 - 5x^4$.

Exercise 21 - Justifying inflection points

A function has $f''(x) = (x-1)(x+2)^2$. Find the inflection points of f and justify.