

AP Calculus AB - Concavity and the Second Derivative - Test Answer Key

Common Core AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9 · maths-pdf.fr/en/

Use this answer key to score the concavity test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9**

Question 1 - Concavity intervals [2 pts]

$$f' = 4x^3 - 24x \text{ and } f'' = 12x^2 - 24 = 12(x^2 - 2).$$

Zeros: $x = \pm\sqrt{2}$. Positive for $|x| > \sqrt{2}$, negative between.

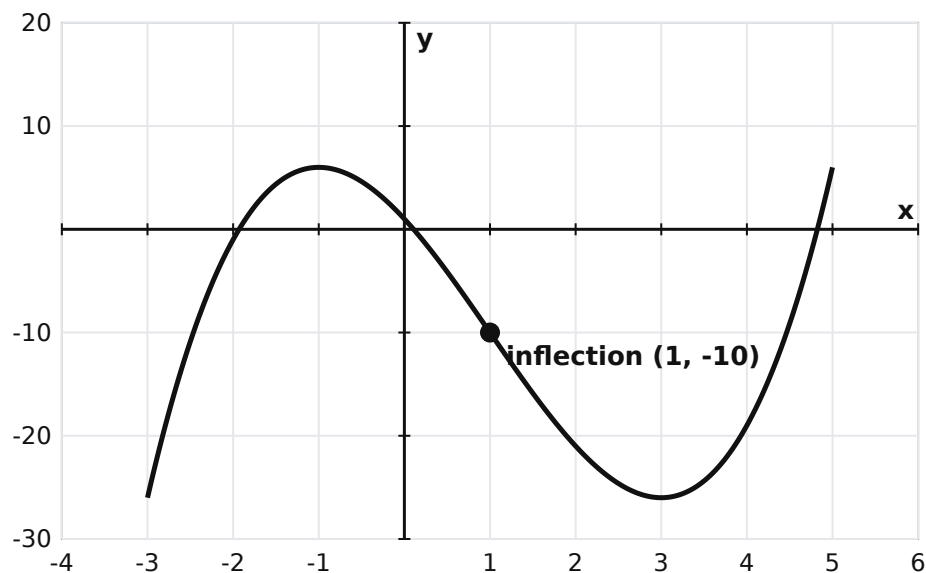
Final answer: **concave up on $(-\infty, -\sqrt{2})$ and $(\sqrt{2}, \infty)$; concave down on $(-\sqrt{2}, \sqrt{2})$.**

Scoring: 1 point for f' ; 1 point for the intervals.

Question 2 - Inflection point [3 pts]

$$f'' = 6x - 6, \text{ zero at } x = 1. f'' < 0 \text{ for } x < 1 \text{ and } f'' > 0 \text{ for } x > 1. f(1) = 1 - 3 - 9 + 1 = -10.$$

Final answer: $(1, -10)$.



Graph of $f(x) = x^3 - 3x^2 - 9x + 1$.

Scoring: 1 point for $f' = 0$; 1 point for the sign change; 1 point for the point.

Question 3 - Second derivative test [2 pts]

$f' = 3x^2 - 3 = 0$ gives $x = \pm 1$. $f'' = 6x$.

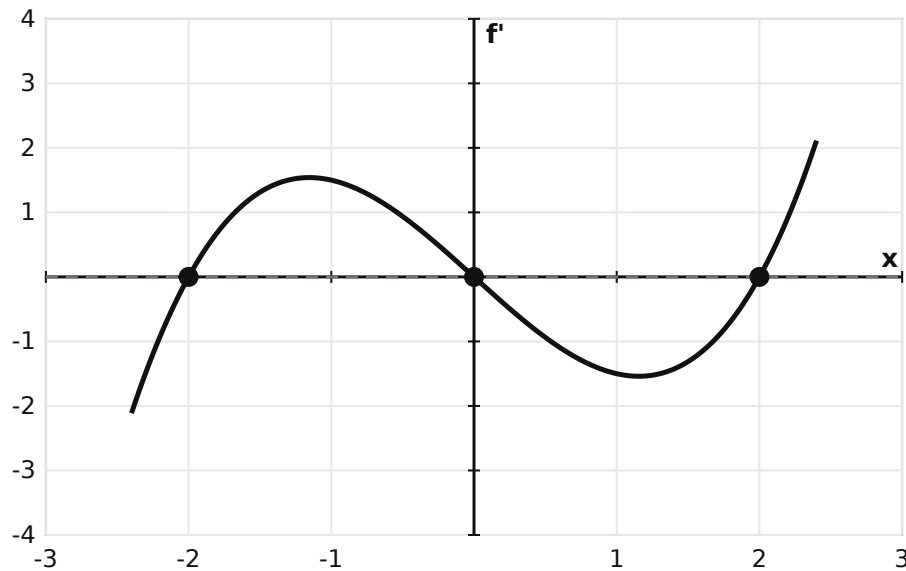
$f''(1) = 6 > 0$: relative minimum, $f(1) = 0$. $f''(-1) = -6 < 0$: relative maximum, $f(-1) = 4$.

Scoring: 1 point for the critical numbers; 1 point for the classification.

Question 4 - Graph of the derivative [3 pts]

$f' = \frac{x^3 - 4x}{2}$, so $f'' = \frac{3x^2 - 4}{2}$, zero at $x = \pm \frac{2}{\sqrt{3}}$.

- Relative minima at $x = -2$ and $x = 2$ (f' goes $-$ to $+$); relative maximum at $x = 0$ ($+$ to $-$).
- Concave up where f' increases: $(-\infty, -\frac{2}{\sqrt{3}})$ and $(\frac{2}{\sqrt{3}}, \infty)$.
- $x = \pm \frac{2}{\sqrt{3}} \approx \pm 1.155$.



The graph of f' (not f).

Scoring: 1 point per part.

Question 5 - Reading a summary table [3 pts]

- On $(-\infty, 1)$ ($f' > 0$).
- At $x = 1$ (f' changes from $+$ to $-$).
- At $x = 3$ (f'' changes from $-$ to $+$).

Scoring: 1 point per part.

Question 6 - Justify a statement [2 pts]

$g'' = 12x^2 \geq 0$ for all x , so the sign of g'' does not change at 0.

Final answer: **No; there is no change of concavity, so there is no inflection point.**

Scoring: 1 point for examining the sign of g'' ; 1 point for the conclusion.

Question 7 - Fastest growth [3 pts]

- $V' = -3t^2 + 24t$ and $V'' = -6t + 24$.
- $V'' = 0$ at $t = 4$, changing from positive to negative: the rate is maximal at **$t = 4$ days**.
- $V'(4) = -48 + 96 = 48$ thousand gallons per day.



The rate $V'(t)$.

Scoring: 1 point per part.

Question 8 - Finding constants [2 pts]

$f'' = 6x + 2a = 0$ at $x = 1$ gives $a = -3$. $f'(3) = 27 + 6a + b = 0$ gives $27 - 18 + b = 0$, so $b = -9$.

Final answer: $a = -3$, $b = -9$.

Scoring: 1 point for a ; 1 point for b .

Total
20 points