

AP Calculus AB - Concavity and the Second Derivative - Test

Common Core AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

This test checks concavity, inflection points, the Second Derivative Test and connecting the graphs of f , f' and f'' (AP Calculus AB, 5.7, 5.8, 5.9). It has 8 questions worth 20 points in total. Allow 45 minutes, work without notes, and show all work, since justification earns points on the AP exam.

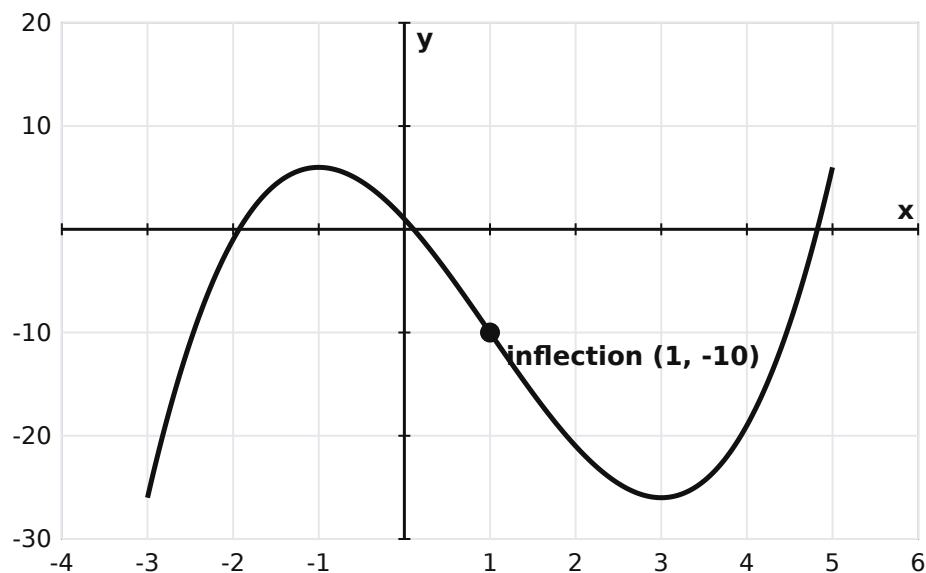
About this test

- Grade: AP Calculus AB
- Chapter: Concavity and the Second Derivative
- Standards: AP CALC AB 5.7, AP CALC AB 5.8, AP CALC AB 5.9
- Time: 45 minutes
- Total: 20 points

Question 1 - Concavity intervals [2 pts]

Find the intervals where $f(x) = x^4 - 12x^2 + 5$ is concave up and concave down.

Question 2 - Inflection point [3 pts]



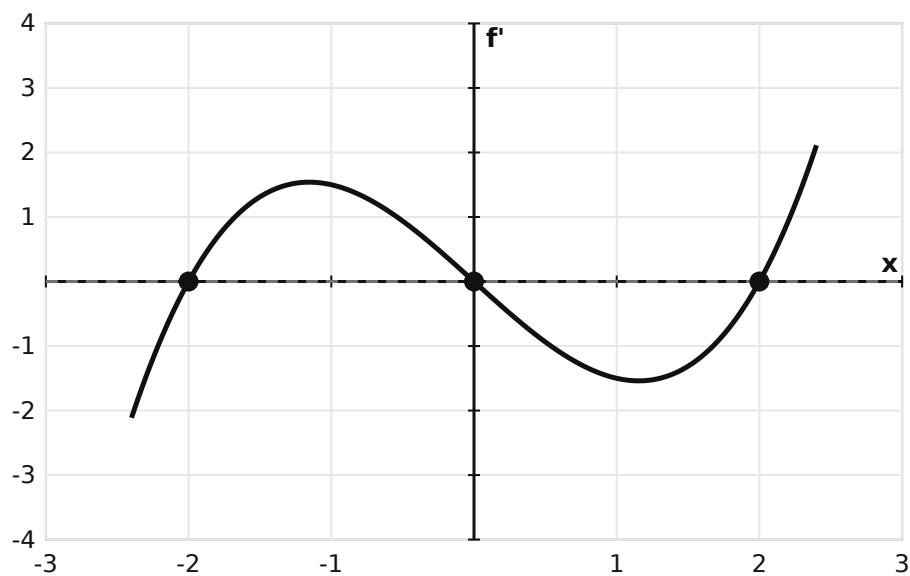
Graph of $f(x) = x^3 - 3x^2 - 9x + 1$.

Find the inflection point of $f(x) = x^3 - 3x^2 - 9x + 1$ and justify with the sign of f'' .

Question 3 - Second derivative test [2 pts]

Use the Second Derivative Test to classify the critical numbers of $f(x) = x^3 - 3x + 2$.

Question 4 - Graph of the derivative [3 pts]



The graph of f' (not f).

The graph shows $f'(x) = \frac{x(x-2)(x+2)}{2}$. (a) Where does f have relative extrema? (1 point) (b) On

which intervals is f concave up? (1 point) (c) Find the x -coordinates of the inflection points. (1 point)

Question 5 - Reading a summary table [3 pts]

interval	$(-\infty, 1)$	$(1, 3)$	$(3, \infty)$
f'	+	-	-
f''	-	-	+

Signs of f' and f'' for a continuous function f .

Use the table. (a) Where is f increasing? (1 point) (b) Where does f have a relative maximum? (1 point) (c) Where is there an inflection point? (1 point)

Question 6 - Justify a statement [2 pts]

Jamal says that $g(x) = x^4$ has an inflection point at $x=0$ because $g''(0)=0$. Is he correct? Explain.

Question 7 - Fastest growth [3 pts]



The rate $V'(t)$.

The volume of water in a reservoir in thousands of gallons is $V(t) = -t^3 + 12t^2$ for $0 \leq t \leq 9$ days.

(a) Find $V'(t)$ and $V''(t)$. (1 point) (b) When does the volume increase fastest? (1 point) (c)

What is that maximum rate? (1 point)

Question 8 - Finding constants [2 pts]

The function $f(x) = x^3 + ax^2 + bx$ has an inflection point at $x = 1$ and a relative minimum at $x = 3$. Find a and b .