

AP Calculus AB - Continuity and the Intermediate Value Theorem - Lesson

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A function is continuous at $x = a$ when $f(a)$ is defined, the limit as x approaches a exists, and the limit equals $f(a)$. Discontinuities are removable (a hole), jump, or infinite. The Intermediate Value Theorem says a function continuous on $[a, b]$ takes every value between $f(a)$ and $f(b)$.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.10, AP CALC AB 1.11, AP CALC AB 1.12, AP CALC AB 1.16**

Key facts

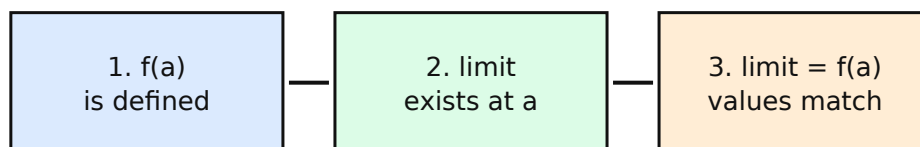
- Continuity needs three things: $f(a)$ exists, the limit exists, and they are equal.
- Removable discontinuities are holes; jump discontinuities have different one-sided limits; infinite discontinuities are vertical asymptotes.
- To make a piecewise function continuous, set the one-sided limits equal at the boundary.
- The IVT guarantees a value, but only if the function is continuous on the whole closed interval.

What does it mean to be continuous?

A function is continuous at a point if you can draw its graph through that point without lifting your pencil. In calculus, this idea is made precise with three conditions.

Continuity at $x = a$

A function f is continuous at $x = a$ if (1) $f(a)$ is defined, (2) $\lim_{x \rightarrow a} f(x)$ exists, and (3) $\lim_{x \rightarrow a} f(x) = f(a)$.

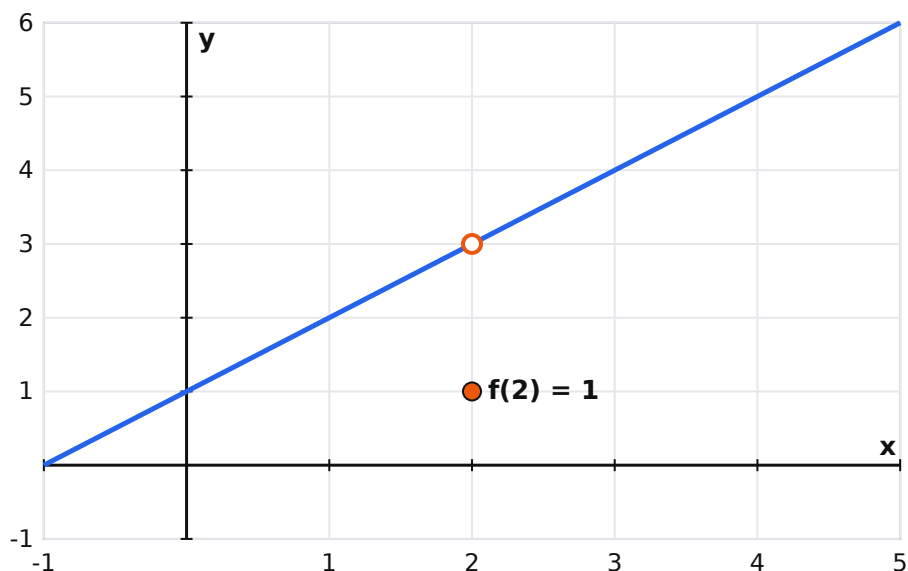


A function is continuous at $x = a$ only when all three conditions hold.

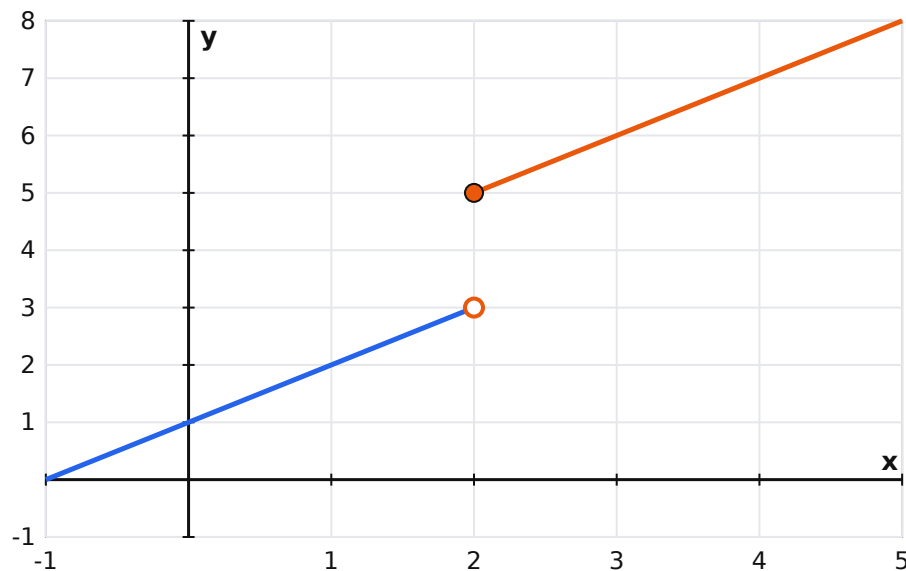
What kinds of discontinuity are there?

Type	What happens	Example
Removable (hole)	The limit exists but $f(a)$ is missing or different.	$\frac{x^2-4}{x-2}$ at $x=2$
Jump	The one-sided limits exist but are different.	A step function such as postage by weight
Infinite	The outputs grow without bound (vertical asymptote).	$\frac{1}{x-3}$ at $x=3$

To classify a discontinuity, simplify first. If a factor cancels, the discontinuity is removable. If a factor stays in the denominator while the numerator is not zero, it is infinite. If the formula changes at a boundary and the two sides disagree, it is a jump.



Removable: the limit is 3 but $f(2) = 1$.



Jump: the one-sided limits are 3 and 5.

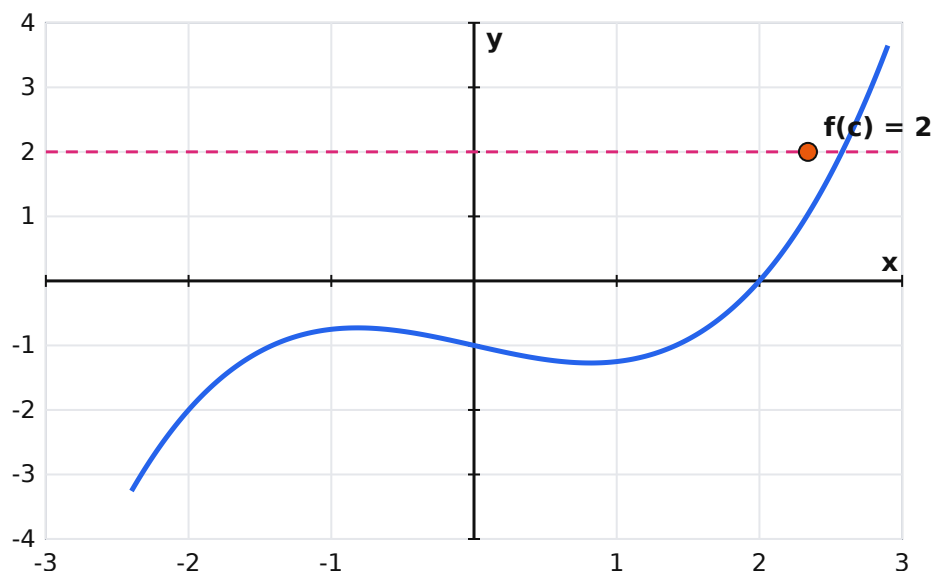
How do you make a piecewise function continuous?

Set the left-hand and right-hand limits equal at the boundary, then solve for the unknown. For example, if $f(x) = x^2$ for $x < 2$ and $f(x) = kx - 2$ for $x \geq 2$, continuity at 2 requires $4 = 2k - 2$, so $k = 3$.

What is the Intermediate Value Theorem?

Intermediate Value Theorem (IVT)

If f is continuous on the closed interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one c in $[a, b]$ with $f(c) = k$.



A continuous graph must cross every height between $f(a)$ and $f(b)$.

To use the theorem on an exam, write three things: f is continuous on the interval, the values $f(a)$ and $f(b)$, and a sentence saying that k lies between them.

Example. Show that $f(x) = x^3 + x - 5$ has a zero in $[1, 2]$. Since f is a polynomial, it is continuous. $f(1) = -3 < 0$ and $f(2) = 5 > 0$, so by the IVT there is a c in $(1, 2)$ with $f(c) = 0$.

Questions about continuity and the intermediate value theorem

What are the three conditions for continuity?

A function is continuous at $x = a$ if $f(a)$ is defined, the limit of $f(x)$ as x approaches a exists, and that limit equals $f(a)$. If any one fails, the function is discontinuous there.

What is a removable discontinuity?

A removable discontinuity is a hole where the limit exists but the function is undefined or has a different value. Redefining the single value at that point repairs the graph.

How can you tell a jump from an infinite discontinuity?

At a jump, both one-sided limits are finite but unequal. At an infinite discontinuity, the outputs grow without bound and the graph has a vertical asymptote.

What does the Intermediate Value Theorem guarantee?

If f is continuous on $[a, b]$ and k is between $f(a)$ and $f(b)$, then $f(c) = k$ for at least one c in $[a, b]$. It proves a value exists but does not tell you where.

Why must f be continuous for the IVT?

A function with a jump can skip over values. For $f(x) = 1/x$ on $[-1, 1]$, f goes from -1 to 1 but never equals 0 because of the asymptote at $x = 0$.